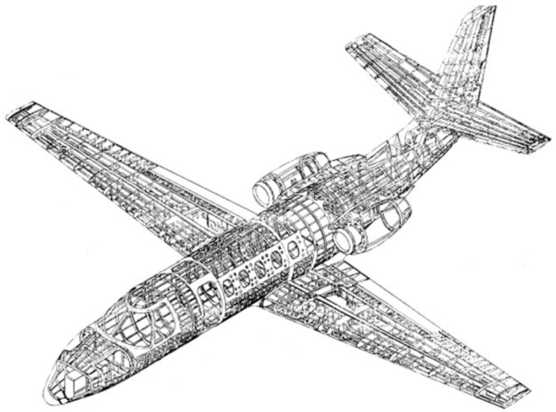

# Exercises from
# A Course on Practical Stress Analysis of Aircraft Structures

# Exercises

# Module 1: Regulations and Global Loading

# Load Factor

Exercise-1A

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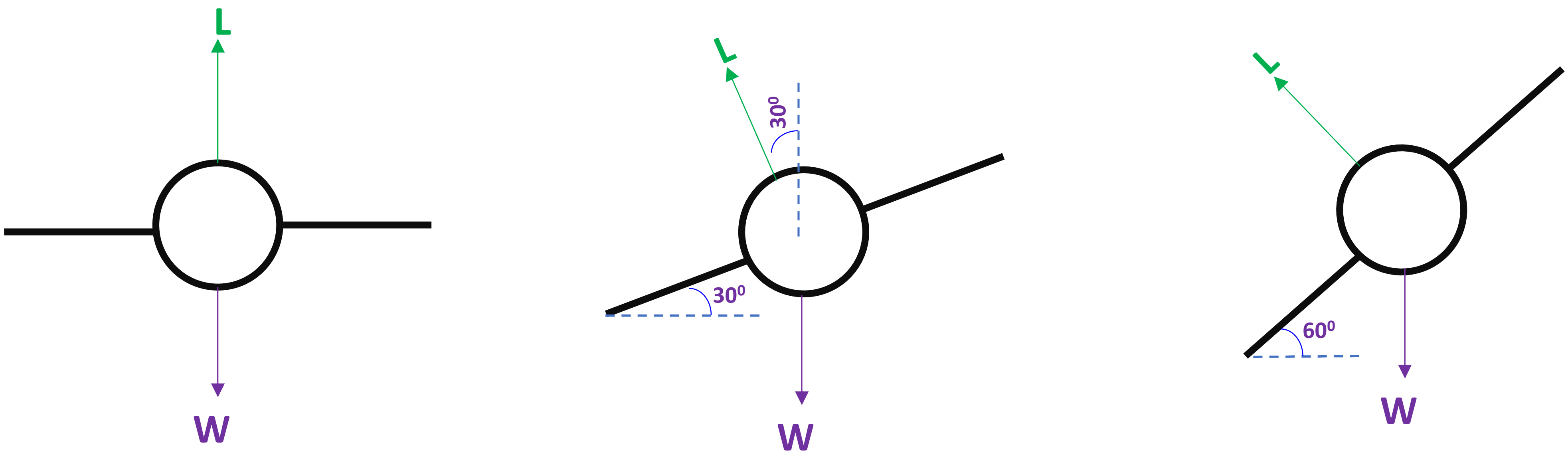
- Calculate the load factor for the following three cases.
- Case 1: Level Flight
- Case 2: Aircraft banking turn at 30deg
- Case 3: Aircraft Banking turn at 60deg

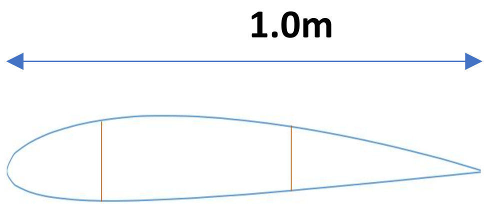


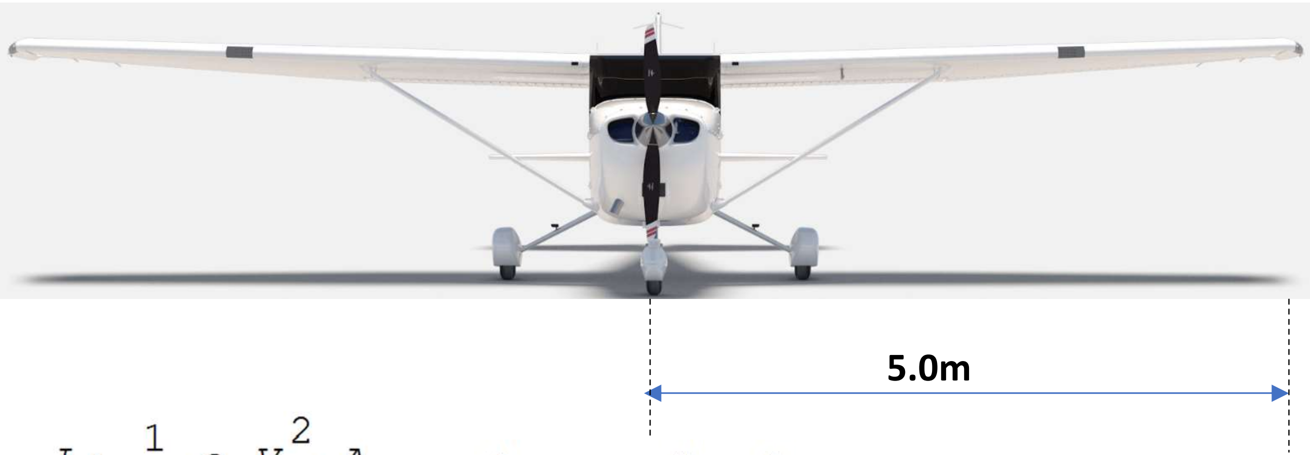
# Aerodynamic Loads





- Calculate the Lift load on the CESSNA 172 wing using the area and standard atmosphere density (ρ = 1.225 kg/m^3) at 120 mph speed.




$$L := \frac{1}{2} \cdot \rho \cdot V^2 \cdot A$$

Assume $C_L = 1$

# Aerodynamic Loads

Exercise-1C

- Calculate the drag load on the antenna using the frontal area and standard atmosphere density (ρ = 1.225 kg/m^3) at 250 mph speed.

**AEROSHIELD SPECIFICATIONS**

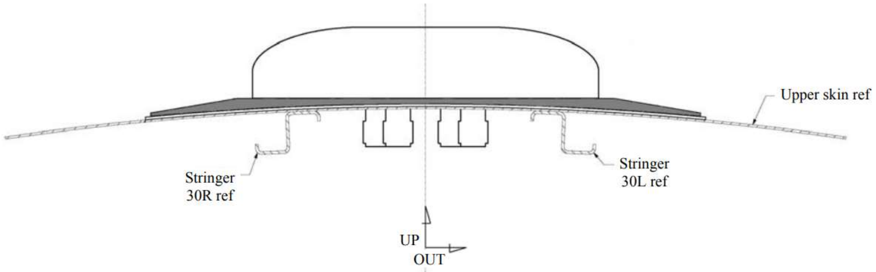
Typical Performance

Weight:

- Radome:
  <56lbs (<25.4Kg).
- Adapter Plate:
  <30lbs (<13.6Kg).

Dimensions:

- Radome:
  15.4"H x 96.1"L x 40.6"W
  39.1cm x 244.1cm x 103.1cm
- Adapter Plate:
  3.2"H x 86.2"L x 39.4"W
  8.1cm x 218.9cm x 100.1cm




Use also the formula for calculating a drag load : Extract from AC 20.43-2b

$$D=0.000327\,AV^2$$

(The formula includes a 90 percent reduction factor for the streamline shape of the antenna.)
D is the drag load on the antenna in lbs.
A is the frontal area of the antenna in sq. ft.
V is the $V_{NE}$ of the aircraft in mph.

# Aerodynamic Loads

Exercise-1D

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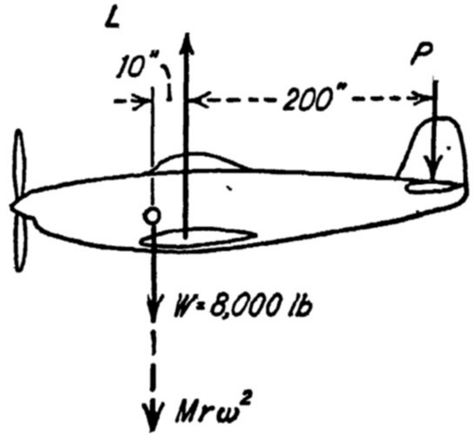
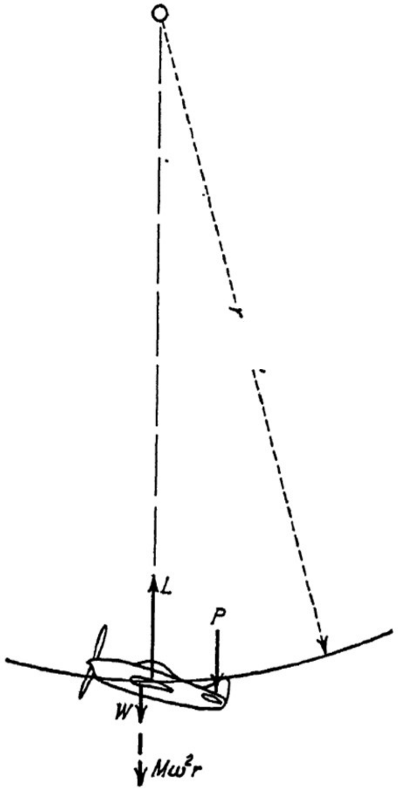
- Calculate the lift force generated by a rotor blade system of following configuration.
- Main rotor has 3 blades. Total diameter of 11m, with aspect ratio of each blade 8
- Consider a rotational speed of 350rpm, CL=1.0 and air density of 1.225 kg/m^3
- Use an average speed equal to 1/3 of blade tip speed for lift calculation

# Manoeuvre Loads

Exercise-1E

- An aeroplane is weighing 8000 lb is flying at 250 mph in a vertical circle with a radius of 595 ft as shown in the figure. Find lift L on the wing and load P on tail at the time airplane is moving horizontally.
- What is the load factor?



# Gust Loads

Exercise-1F

- Determine the gust load factor for an airplane weighing 50000 lbs and a wing area of 1460 square feet when the speed is 250 mph and encounters a gust of 30 fps. Use the slope of lift curve of 4.2
- Equation from CAR 4b

$$n := 1 + \frac{K \cdot U \cdot V \cdot a}{575 \cdot \left(\frac{W}{A}\right)}$$

$$K := 1.33 - \frac{2.67}{\left(\frac{W}{A}\right)^{\frac{3}{4}}}$$

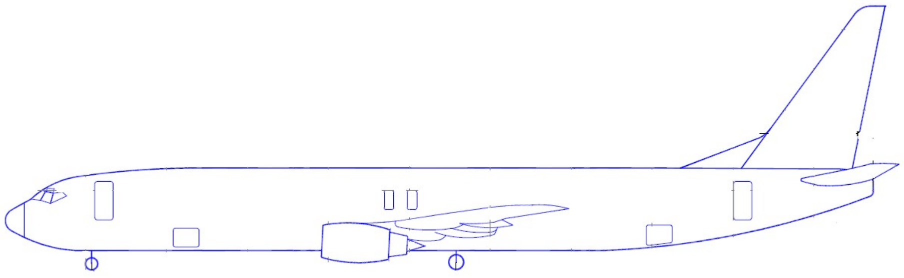
Use the formula derived previously and calculate "n" and compare.

$$n = 1 + \frac{\rho \cdot V \cdot A \cdot \frac{\delta C_L}{\delta \alpha} \cdot V_g}{2 \cdot W}$$

# Cabin Pressurisation Loading

Exercise-1G

- Calculate the hoop stress and longitudinal stress for the fuselage of diameter 4m, skin thickness of 1.5mm subjected to a pressure differential of 11 psi. Consider additional factor of 1.33 as per CS25.365 and a factor of safety of 1.5 as per CS25.303



- Assess the impact of stringers and frames in reducing these stresses.

Additional Exercise

# Bird Strike

Exercise-1H

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- Estimate the impact force due to a bird (m=2.2 kg) hitting on an antenna at a relative velocity of 200 kmph. Consider the impact duration of 0.01 sec.

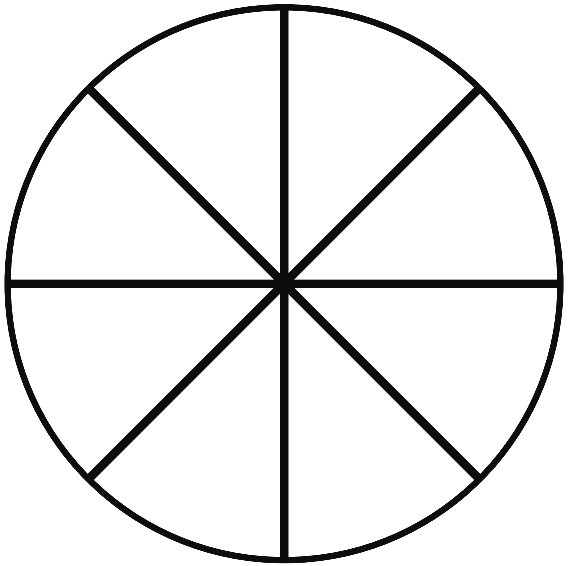
# Thermal Loads

Exercise-1I

- A composite vertical tail has a metallic front spar whose length is 2 m. Using thermal coefficient of $20x10^{-6}$ for spar and 0 for the composite skin, evaluate the stresses that will be developed for the operating temperatures of $-30^0$ C (build temperature of $20^0$ C)

# Cargo and Baggage Loads

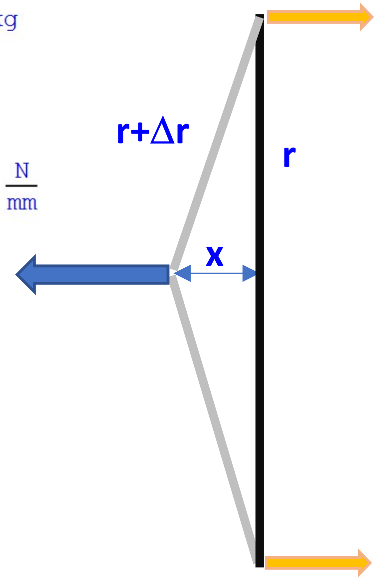
- A cargo net is attached at 8 points to the fuselage of radius 2m, equally spaced around the perimeter. A load of 300 kg is to be supported by this net. If the stiffness of the rope is 7000 N/mm, calculate the forces developed at each attachment for a forward load in 9g condition.



$m := 300\ \text{kg}$

$r := 2\ \text{m}$

$k := 7000\ \frac{\text{N}}{\text{mm}}$



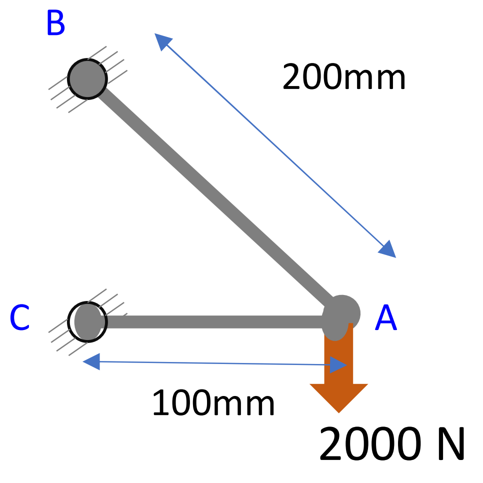

# Exercises

# Module 2: Interface Loads

# Freebody Diagrams

Exercise – 2A

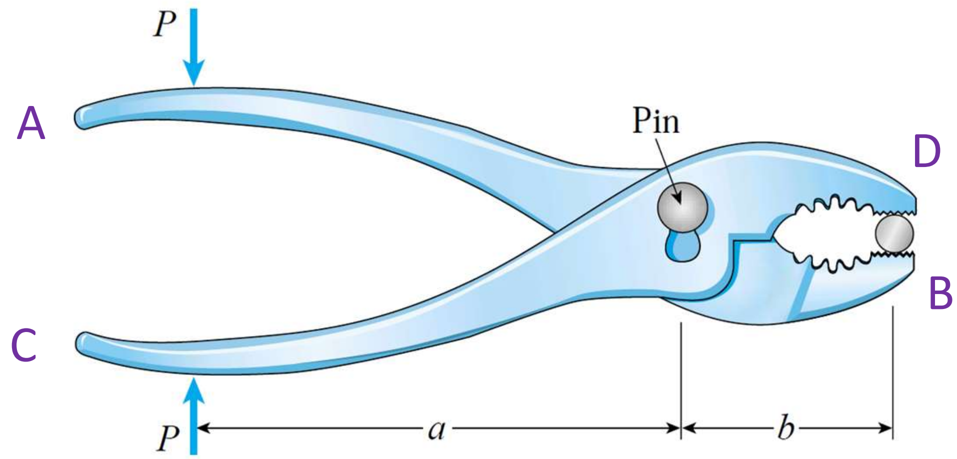
- Draw the freebody diagram of Bars AB and AC and calculate the forces in each member.



# Freebody Diagrams

Exercise – 2B

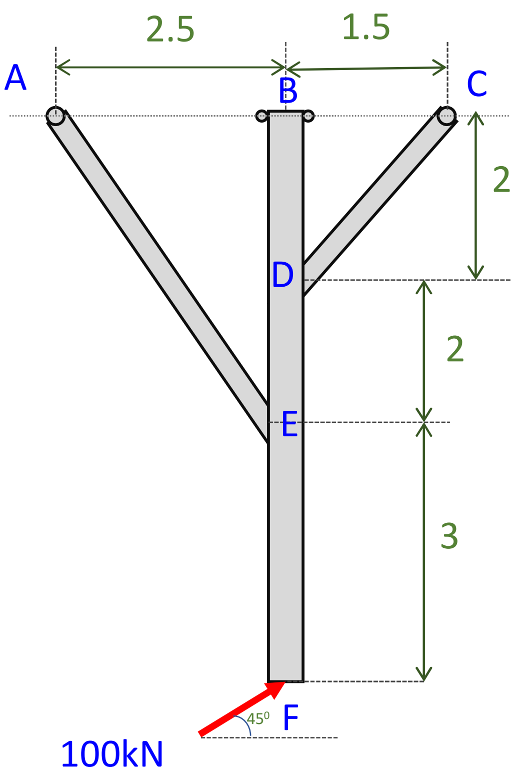
- Draw the freebody diagram
- What is the clamping force ?
- What is the force on the pin ?
- Equilibrium in 3D space ?



# Freebody Diagrams

Exercise – 2C

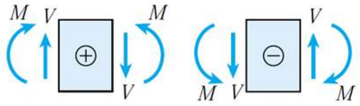
- Find out the reactions at A, B and C. Treat A,C,D & F as pinned joints and B is a guided joint.

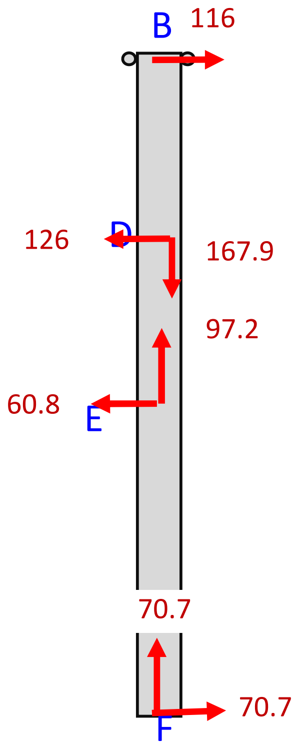


# SF and BM

Exercise – 2D

- Draw SF and BM for the vertical member of the landing gear as shown for the transverse loading.



# Wing SF, BM and Interface Loads

Exercise – 2E

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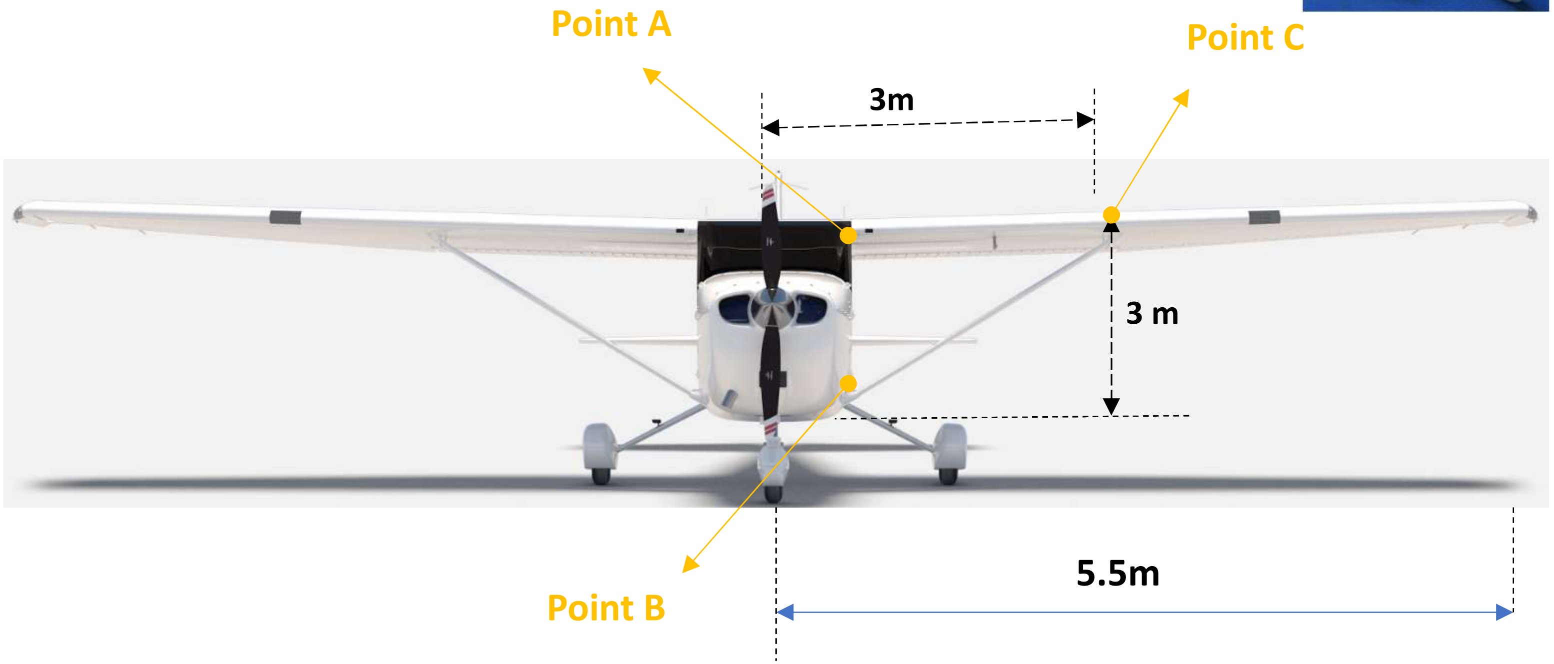
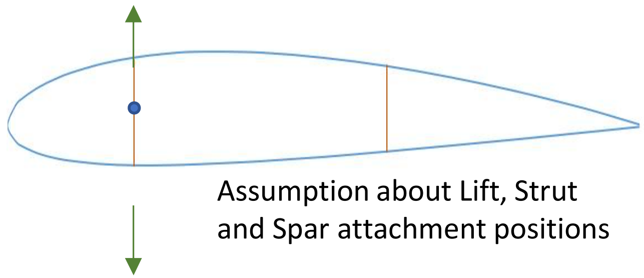
- Calculate the force in the strut of Cessna 172 for the flight condition with vertical load factor of 4.4g.
- A/C weight of 1200 kg and lift distribution is uniform along the length of the wing. Use Safety factor of 1.5.
- Spar is connected by one lug. Both ends of strut are lug connections. Point A can transfer force in both directions.



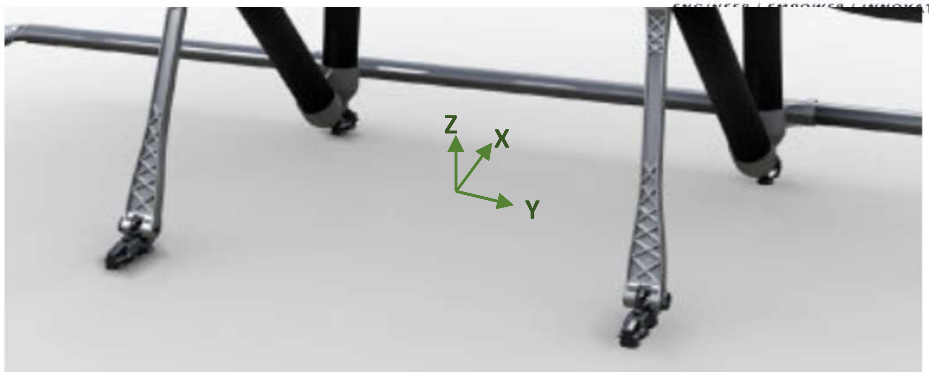



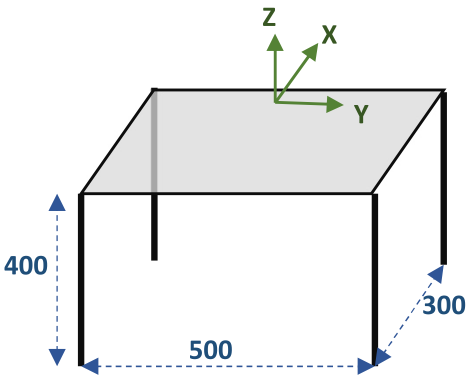
Assumption about Lift, Strut and Spar attachment positions

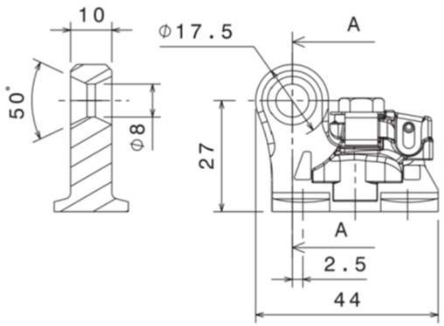
# Bolt Group Analysis

Exercise – 2F

- Calculate the fitting loads for the configuration shown below.
- Dynamic forward load condition of 25.562
- Add 20kg for each seat.
- Calculate the margin if the allowable load is 45kN



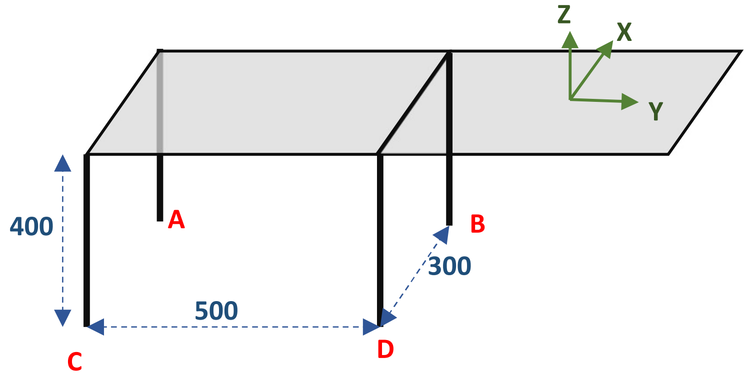




Ultimate load: *
$F_{XZ45°}$ > 45.0 kN / 10100 lbf

# Bolt Group Analysis

Exercise – 2G

- Calculate the fitting loads for the configuration shown below.
- Mass of 100 Kg located in one seat. 16g forward loading.
- Which fitting is highest loaded ?



# Displacement Compatibility

Exercise – 2H

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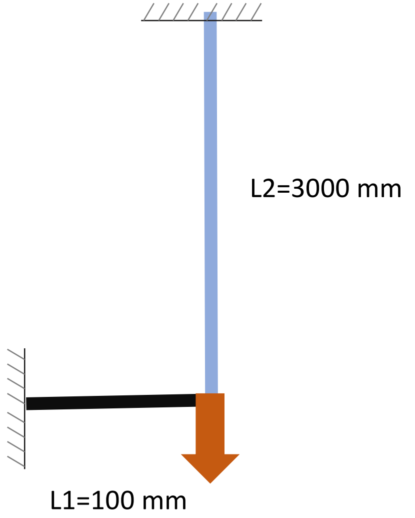
- Calculate the forces reacted by the members 1 and 2.
- Cross section of both members is 20x20 mm
- Applied load = 20 kN



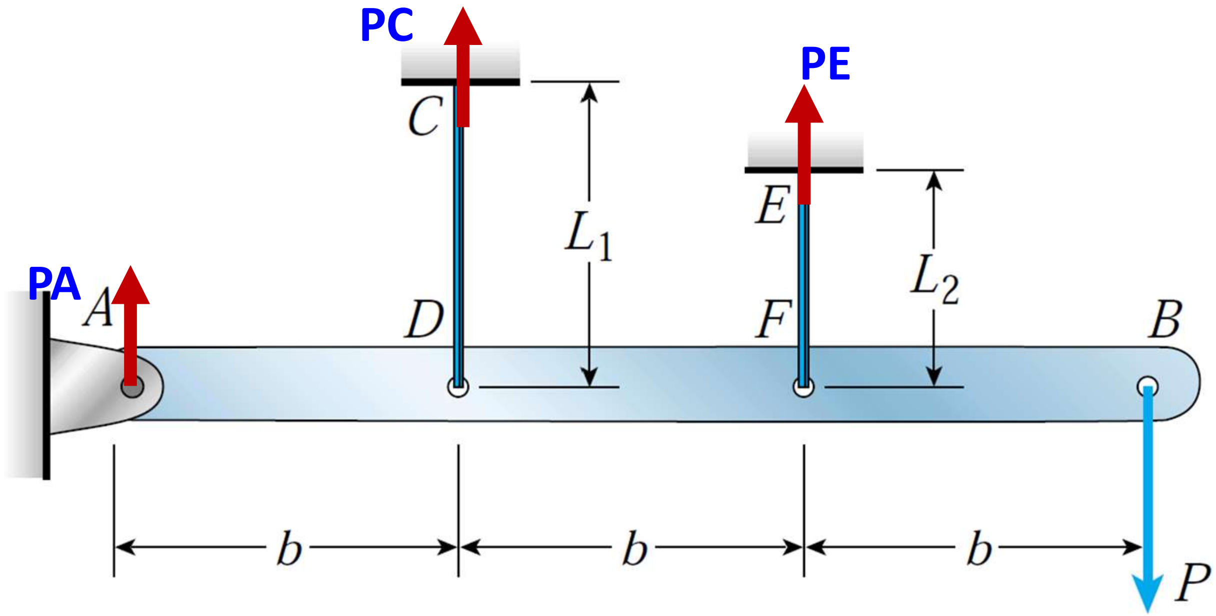

$L1 := 100\ mm$

$L2 := 3000\ mm$

# Displacement Compatibility

Exercise – 2I

- Derive the formula for the load share at Points A, C and E, consider L1 = L2, CD and EF members are made of same material and cross section. AB is rigid member

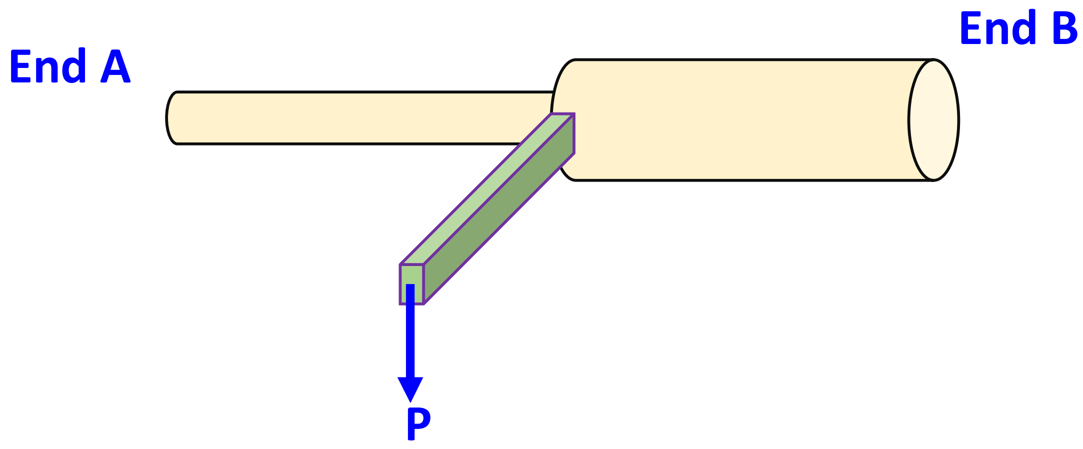


# Displacement Compatibility

Exercise – 2J

- Derive the formula for torsional moments reacted at both ends if the radii of the rods is "r" and "2r"

$$\frac{\tau}{r} = \frac{G \cdot \phi}{L} = \frac{T}{J}$$



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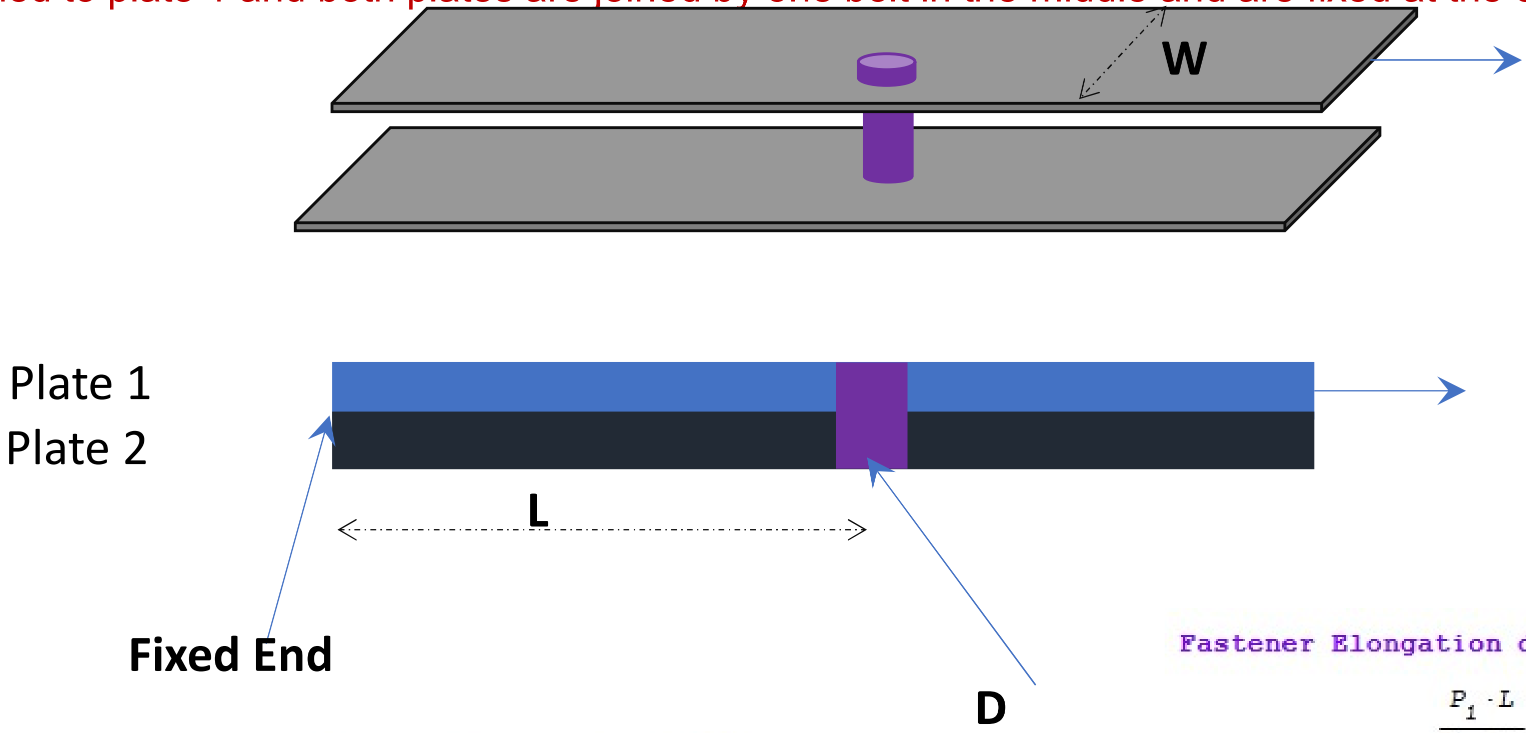
# Displacement Compatibility

- Calculate the load taken by the bolt for the following connection.
- Load is applied to plate 1 and both plates are joined by one bolt in the middle and are fixed at the other end.




Applied Load $P := 1000\ N$

$L := 30\ mm$ $D := 5\ mm$ $W := 20\ mm$

$t_1 := 3\ mm$ $E_1 := 70000\ MPa$ $t_1 := 3\ mm$

$t_2 := 3\ mm$ $E_2 := 70000\ MPa$ $t_2 := 3\ mm$

$E_f := 70000\ MPa$

$\upsilon_f := 0.33$

Fastener Elongation due to Pure Shear

$$\frac{P_1 \cdot L}{A_1 \cdot E_1} = \frac{P_2 \cdot L}{A_2 \cdot E_2} + \frac{P_2 \cdot t}{A_f \cdot G_f}$$

$$P_1 + P_2 = P$$

Fastener Elongation due to Bending

$$\frac{P_1 \cdot L}{A_1 \cdot E_1} = \frac{P_2 \cdot L}{A_2 \cdot E_2} + \frac{P_2 \cdot t^3}{3 \cdot E_f \cdot I_f}$$

$$P_1 + P_2 = P$$

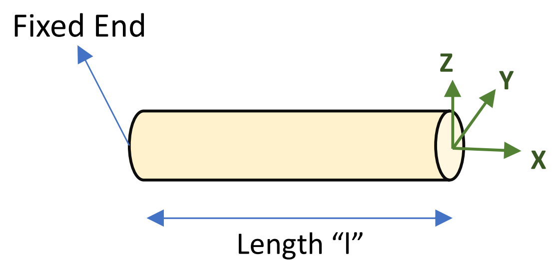
# Exercises

# Module 3: Strength Analysis

# Types of Stresses

Exercise – 3A

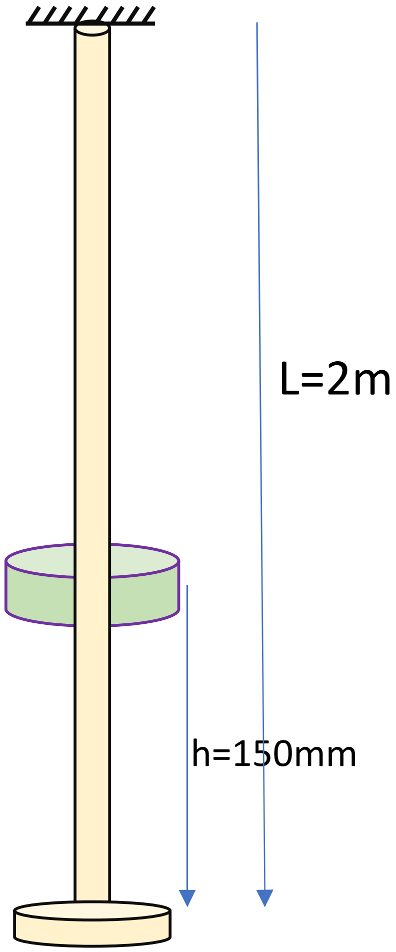
- Calculate the stresses for on the bar of circular cross section for the following loads/moments.
- Radius = 20 mm, Length = 100 mm
- Px = 10 kN, Py=5 kN, Mx=2 kN.m



# Axial Loading

Exercise – 3B

- Calculate the elongation of rod (steel E=210GPa) when the block falls onto the lower flange as shown without rebound.
- Rod diameter = 16 mm, Mass of block = 25 kg
- What is the equivalent static load

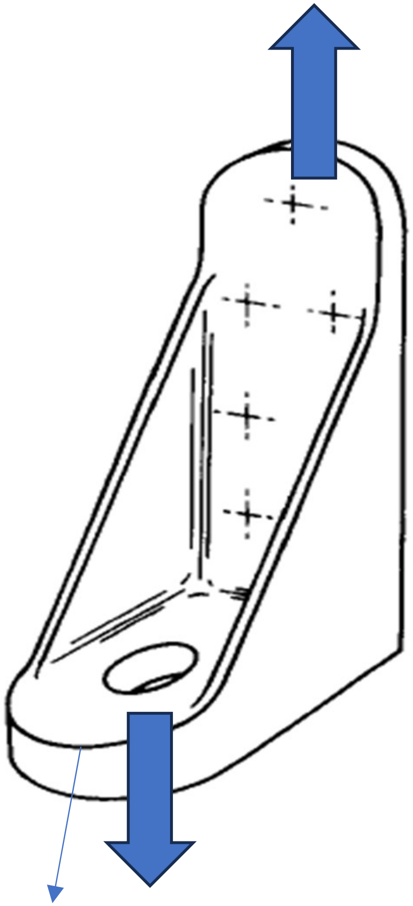


# Shear Stresses



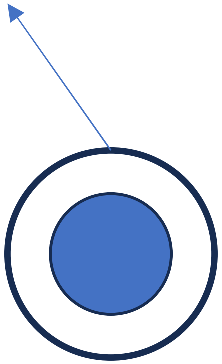


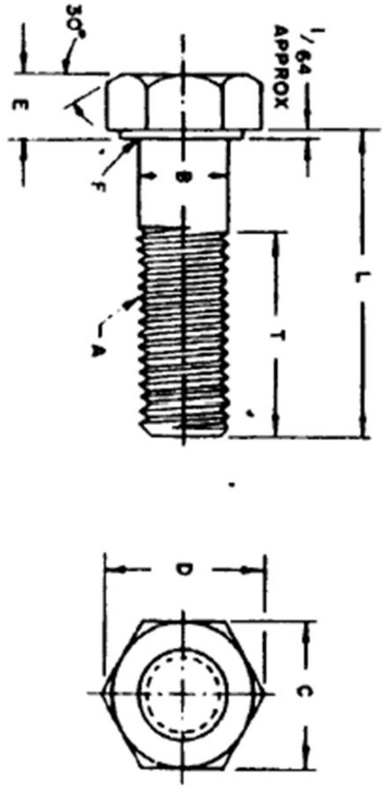
- Calculate the shear stresses on the flange A, thickness of which is 10mm. Connection is by ½ in dia Bolt MS16208. Applied load is 50kN.



Flange A

Consider 1.5d





# Torsion Loading– Circular members

Exercise – 3D

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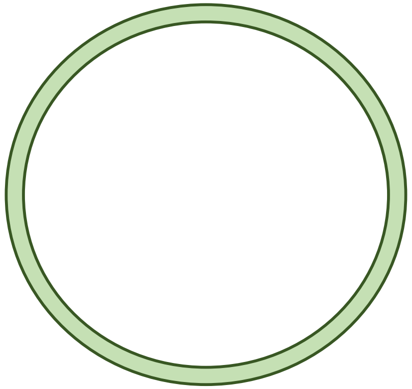
- Derive the formula for maximum shear stress in the tube subjected to torsion loading. Thickness is very small compared to radius

$$\frac{\tau}{r} = \frac{T}{J}$$

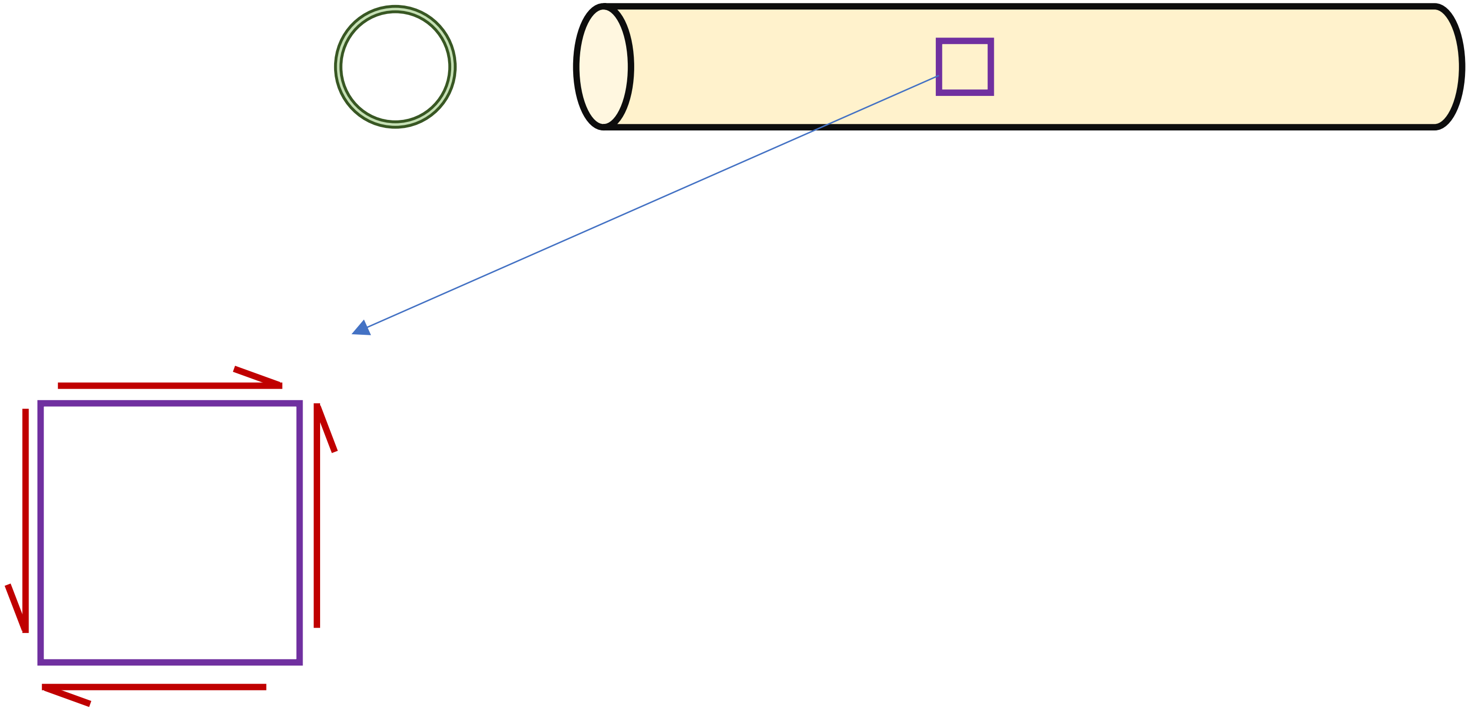
$$J = \frac{\pi \cdot r^4}{2}$$



# Torsion Loading – Inclined plane

Exercise – 3E

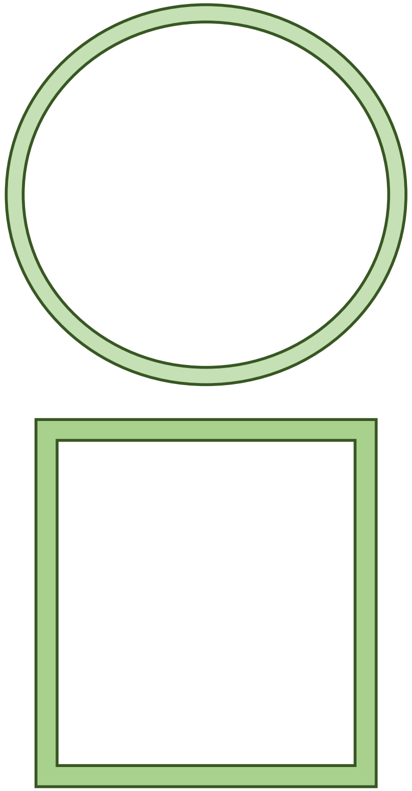
- Derive the formula for stresses in the inclined plane of tube subjected to torsion loading.



# Torsion - Thin walled tubes

Exercise – 3F

- Calculate the maximum stress for the following two cross sections
- r=50mm, t=3mm and the square section has the same thickness and cross section area.
- Torque of 20 KN*m, G=150 Gpa



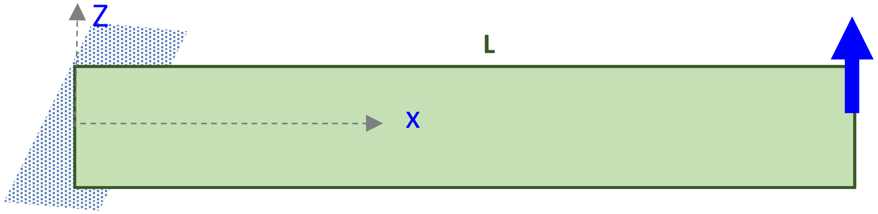
Additional Exercise

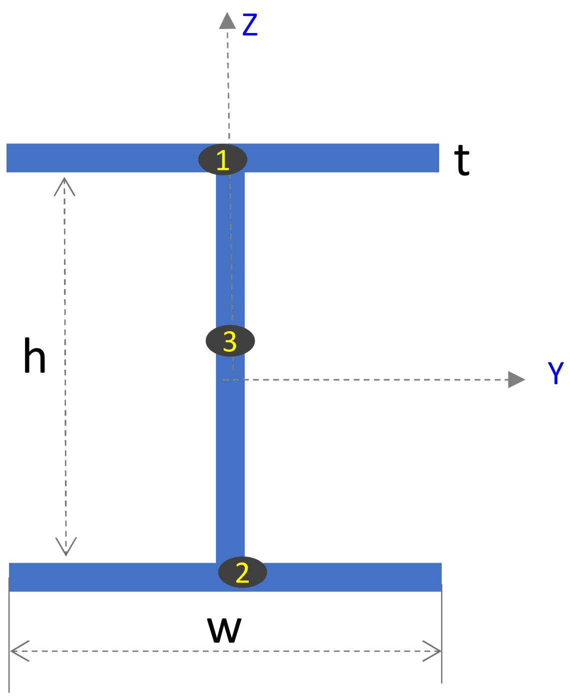
Calculate the angle of twist per unit length for the two cross sections

# Symmetrical Members in Bending

Exercise – 3G

- Calculate maximum deflection, Bending and Shear stress for the following Symmetrical I section under applied bending load due to 5000 N for the beam length of 500 mm. w=40 mm, h=60 mm, t=3 mm



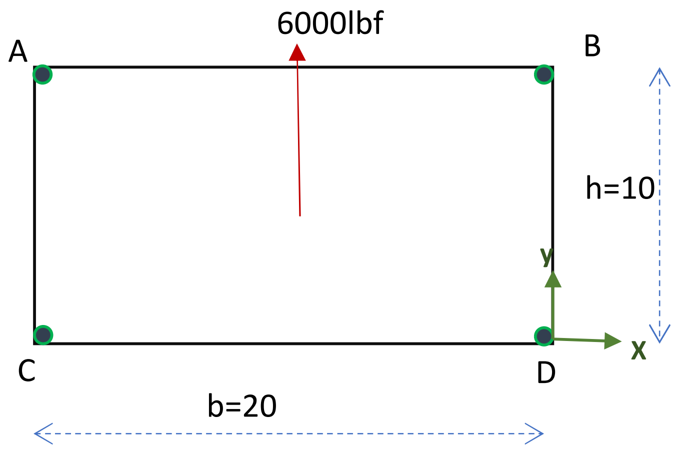



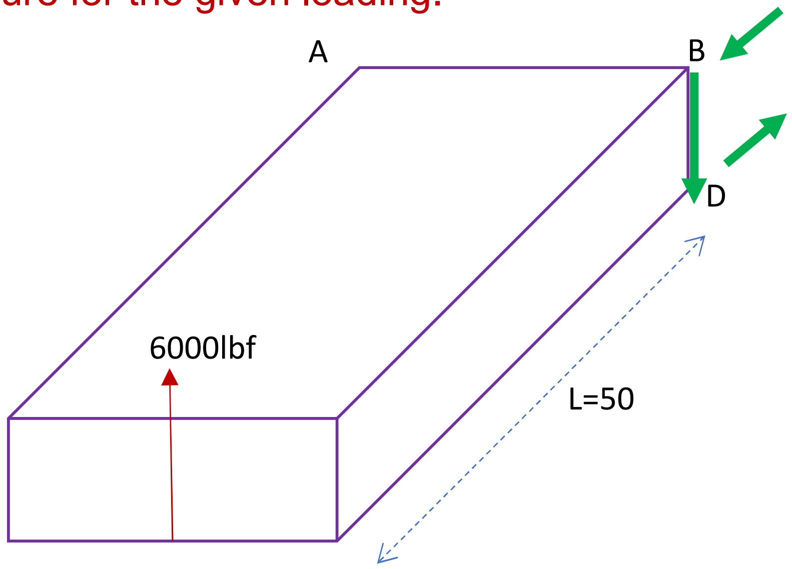

Additional Exercise

Repeat the calculations for P in Y direction

# Symmetrical Members in Bending – Direct stress

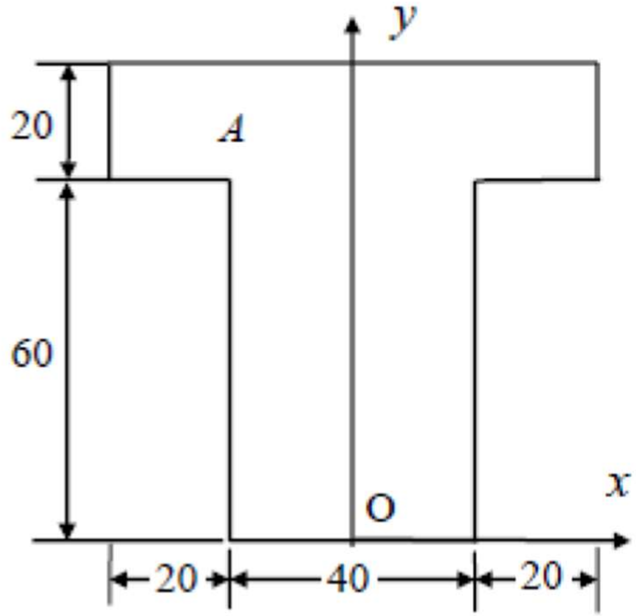
- Calculate the stress in the members A,B,C&D of the idealised structure for the given loading.
- Idealised areas = 1 in^2



# Cross section properties

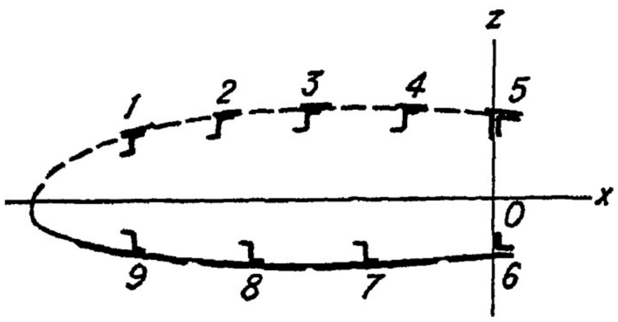
- Calculate CG,I for the following cross section about x axis.

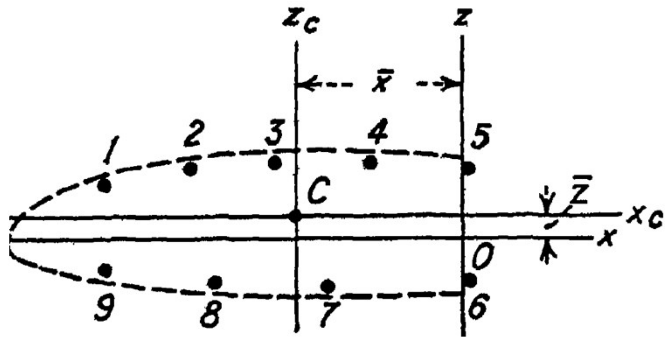


# Cross section properties

Exercise – 3J

- Calculate the Ixx, Iyy and Ixy



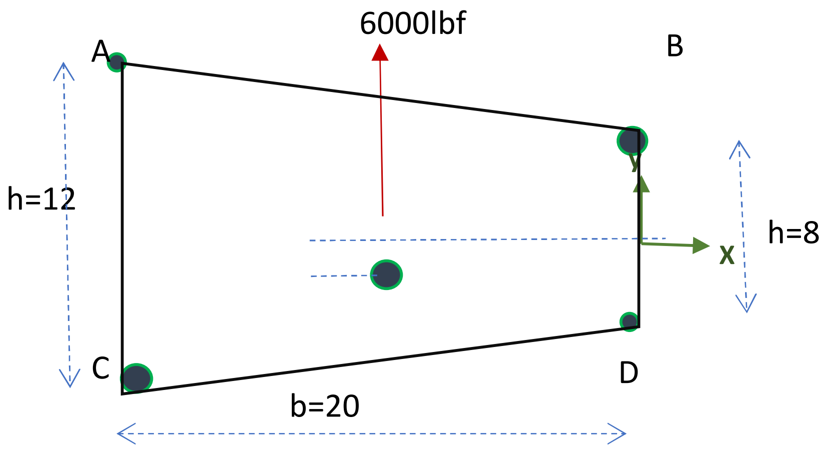



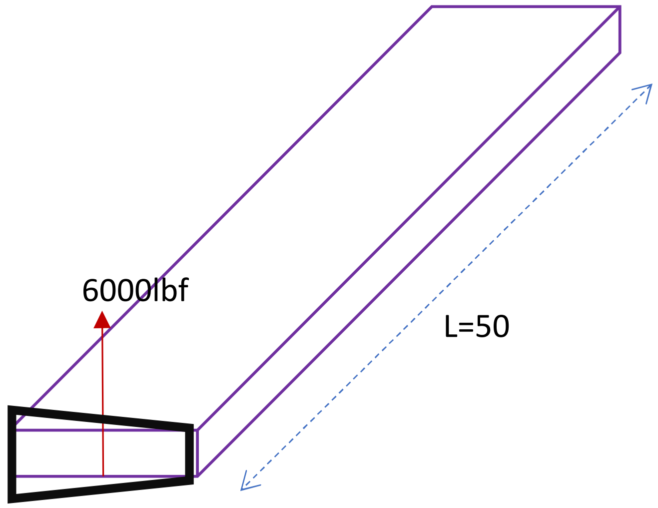

| Element (1) | A (2) | x (3) | z (6) |
|---|---|---|---|
| 1 | 0.358 | −34.5 | +8.6 |
| 2 | 0.204 | −28.1 | +9.6 |
| 3 | 0.395 | −19.9 | +10.0 |
| 4 | 0.204 | −10.1 | +9.6 |
| 5 | 1.615 | +0.5 | 8.8 |
| 6 | 1.931 | +0.5 | −5.7 |
| 7 | 0.752 | −10.1 | −5.2 |
| 8 | 0.784 | −22.4 | −4.3 |
| 9 | 0.892 | −34.7 | −2.4 |

# Unsymmetrical Members in Bending – Direct stress

Exercise – 3K

- Calculate the stress in the members A,B,C&D of the idealised structure for the given loading.
- Idealised areas A,D= 1 , B,C=2 in^2




$P_y := 6000\ \text{lbf}$

$L := 50\ \text{in}$

$M_x := P_y \cdot L = 300000\ \text{lbf in}$

$P_z := -0\ \text{lbf}$

$L := 50\ \text{in}$

$M_y := P_z \cdot L = 0\ \text{lbf in}$

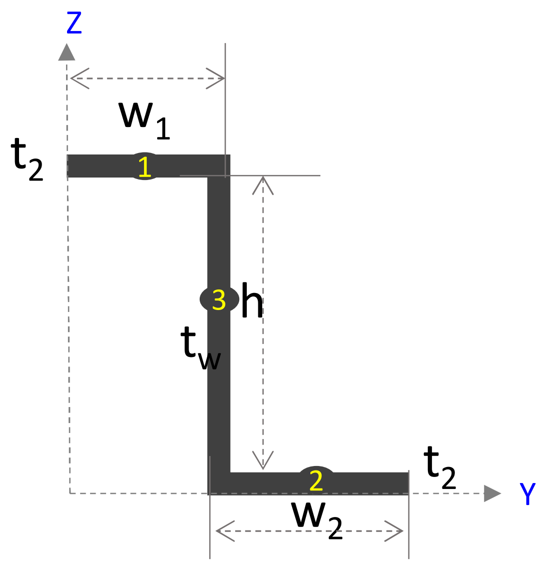
$b := 20\ \text{in}$

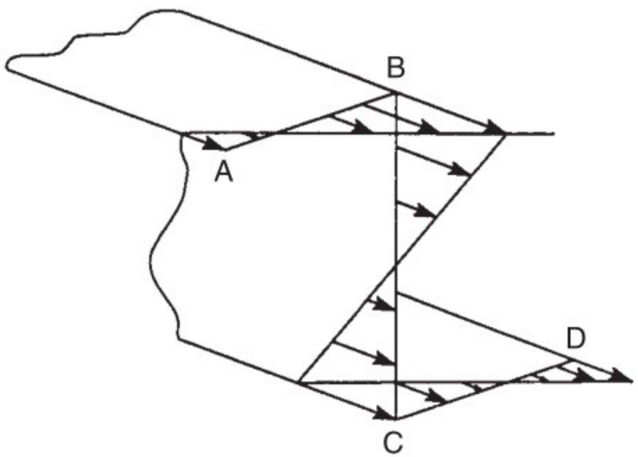
$h := 10\ \text{in}$

# Unsymmetrical Members in Bending – Direct stress

Exercise – 3L

- Calculate maximum deflection, Bending and Shear stress for the following Unsymmetrical Z section under applied bending load due to 5000 N for the beam length of 500 mm. w1=w2=30 mm, h=60 mm, t=3 mm

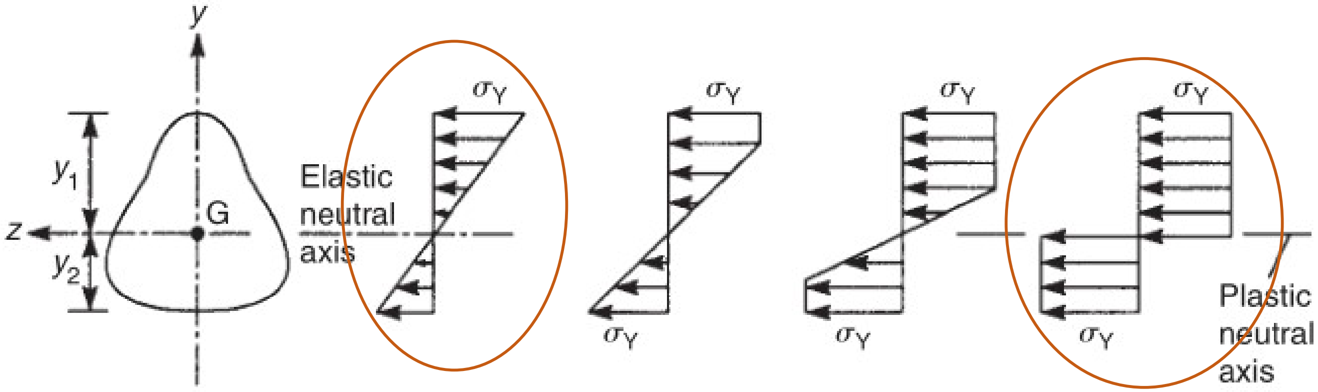


# Plastic Bending

Exercise – 3M

- Derive the ratio of moments that gives raise to stress distribution as shown for Case 1 and Case 2. Assume elastic – plastic stress strain curve and a constant width of “b” and y1=y2 for a symmetrical section about z-axis. Assume a total height of “h”. (h=y1+y2)




Additional Exercise
Circular cross section

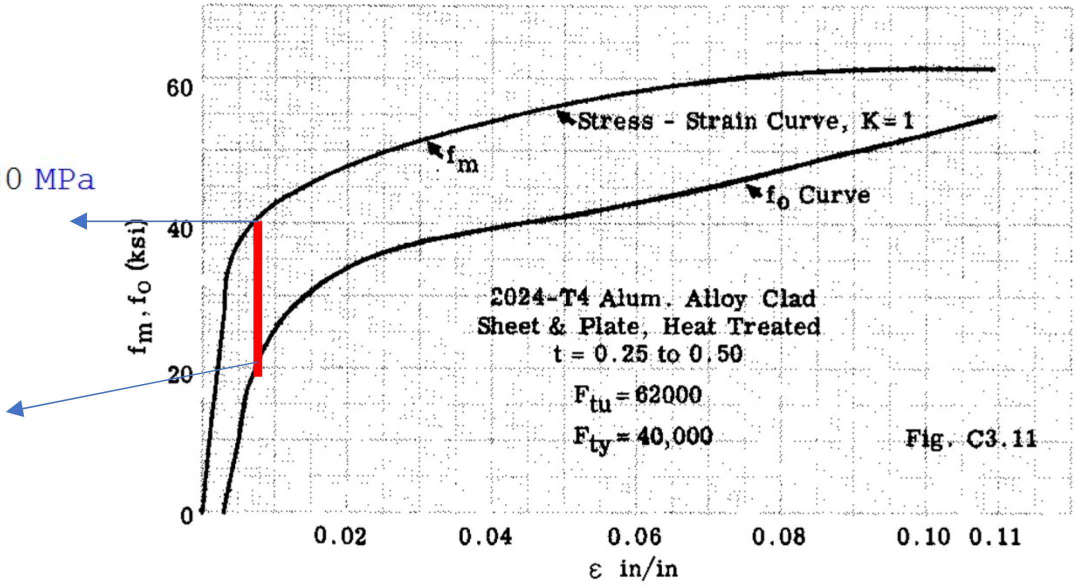
# Plastic Bending

Exercise – 3N_1

- Calculate the allowable moment for the following rectangular cross section , b=20 mm, h=50mm
- Case 1: no yielding allowed
- Case 2: Yielding, using shape factor approach
- Case 3: Yielding, using Cozzone approach

$$\sigma_{all} := 280\ \mathrm{MPa}$$

$$\sigma_B := \sigma_m + \sigma_o \cdot (k - 1)$$

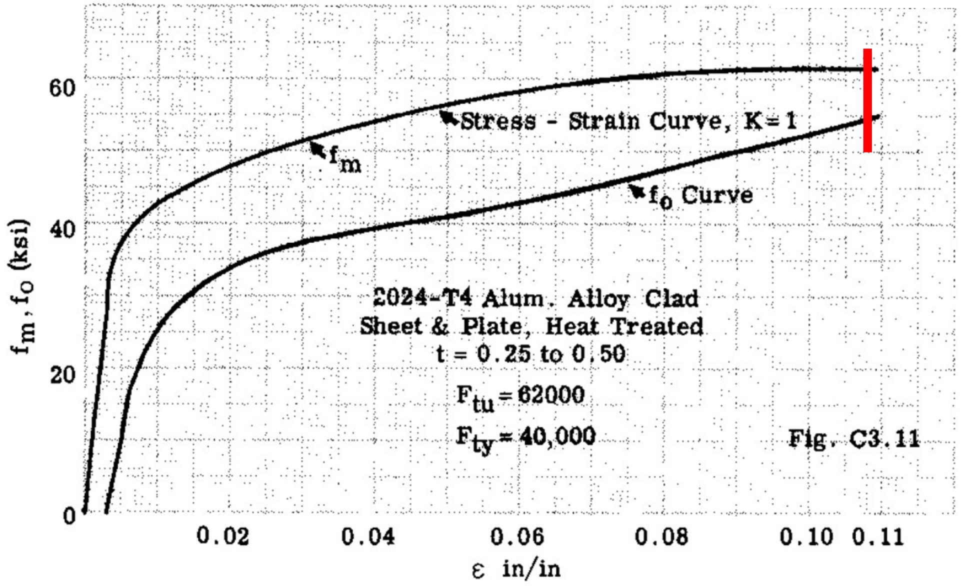
# Plastic Bending

Exercise – 3N_2

- Repeat the same analysis considering ultimate stress.
- Case 1: Maximum stress = Failure stress
- Case 2: Plastic bending , using shape factor approach
- Case 3: Plastic bending, using Cozzone approach


$\sigma_{all} := 414\ MPa$

$414\ MPa = 60\ ksi$



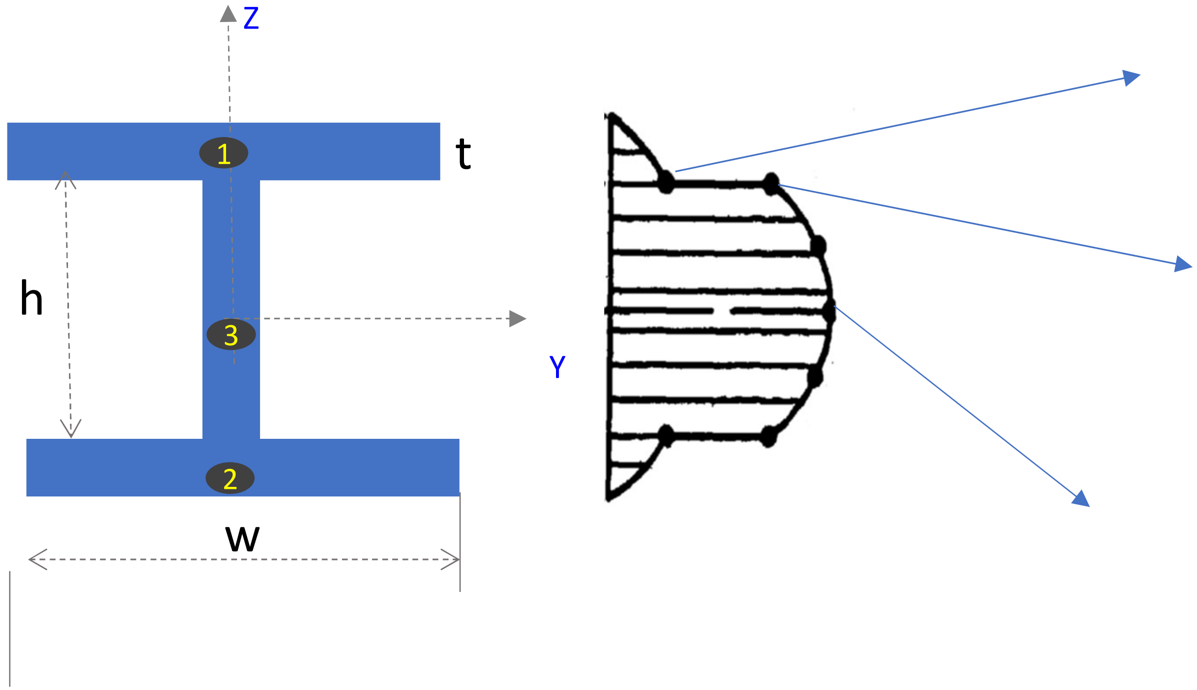
# Symmetrical Members in Bending – Shear Stress

- Calculate the shear stress distribution for the cross section shown, subjected to shear load of 80000 lbf
- h=6, w=4, t=1

$$\tau_{xz} = \frac{P_z \cdot Q}{I_{yy} \cdot t_z} \qquad Q = \int z \, dA$$

$P := 80000\ \text{lbf}$

$h := 6\ \text{in} \qquad w := 4\ \text{in} \qquad t := 1\ \text{in}$



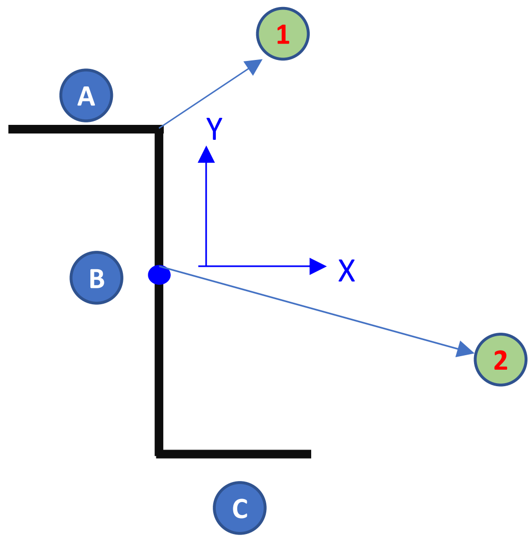
# Unsymmetrical Members in Bending – Shear stress

Exercise – 3P

- Calculate the shear stresses at points 1 and 2 of the below section subject to Py=12000 lbf

$V_x := 0$ lbf

$V_y := 12000$ lbf

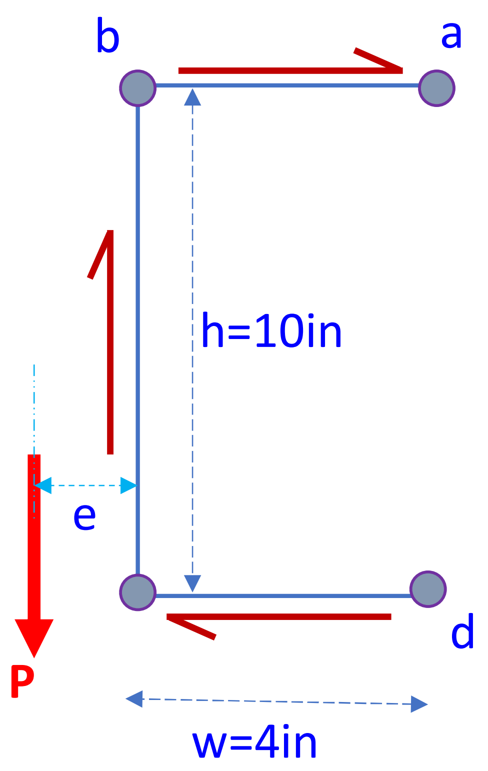



| | Dimension | |
|---|---|---|
| | X | Y |
| **Mem** | **b** | **h** |
| A | 1 | 0.1 |
| B | 0.1 | 3 |
| C | 1 | 0.1 |

# Unsymmetrical Members in Bending – Shear Centre

Exercise – 3Q

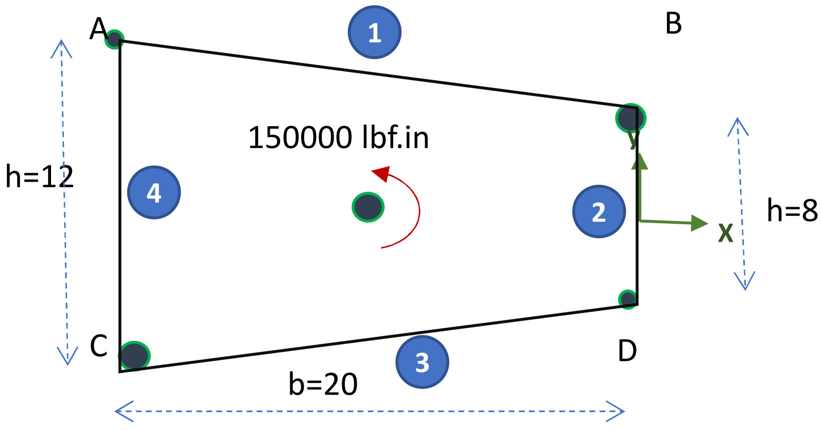
- Find the shear centre position for the cross section shown, all lumped areas are equal to 1 in^2.
- Assume a load of 20000 lbf for calculations



# Torsion - Thin walled tubes – closed sections

- Calculate the shear stresses in the skin and spar of the cross section shown due to pure torsional moment
- Also calculate the twist per unit length if G= 5000 ksi

Exercise – 3R




$G := 5000$ ksi

$T := 150000$ lbf in

$$J = \frac{4A^2}{\int \frac{ds}{t}} = \frac{4A^2}{\sum_{i=1}^{n} \frac{b_i}{t_i}}$$

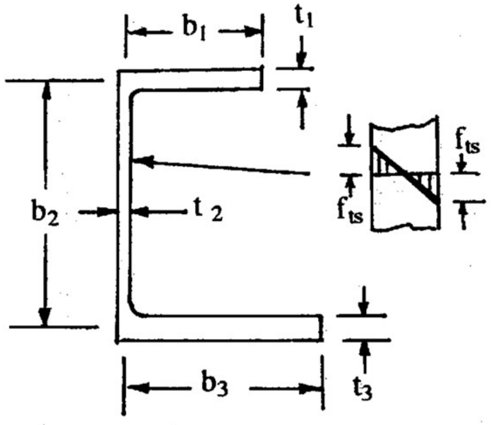
$$J := \frac{4 \cdot A^2}{k}$$

- Calculate stress using conventional formula and compare
- Approximate "r"

$$\frac{\tau}{r} = \frac{T}{J}$$

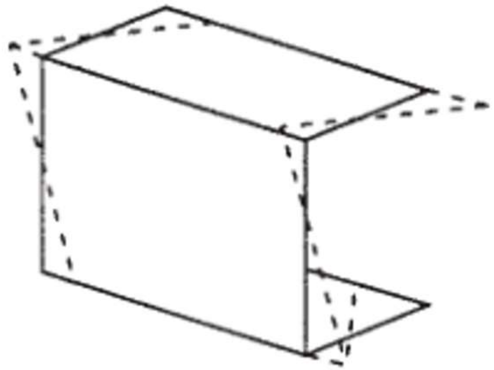
# Torsion Loading – Thin walled members : Open sections

Exercise – 3S




$$f_{ts} = \frac{Tt}{J}$$

$$J = \frac{1}{3}\left(\sum_{i=1}^{n} b_i t_i^3\right)$$

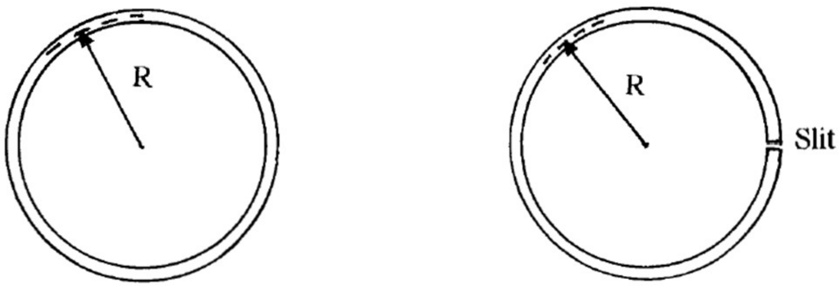


- Calculate "J" if the rectangle section is open and compare it with the closed section of "3P"

# Torsion Loading – Thin walled members : Open sections

- Compare the torsional shear stress and stiffness of the two cross sections shown below. (R/t=20)

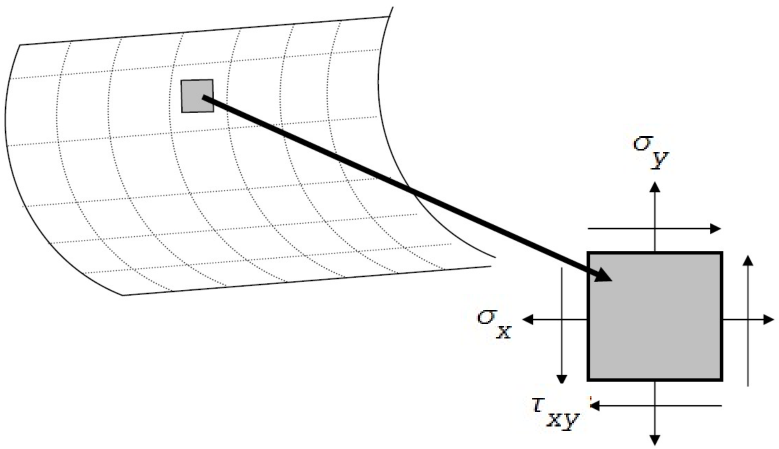
Exercise – 3T



# Combined Stresses

Exercise – 3U

- Calculate maximum principal stress and maximum stress for different combinations of pressurization and torsion of the fuselage skin.
- Hoop Stress due to pressurization = 80 MPa
- Longitudinal Stress due to pressurization = 40 MPa
- Longitudinal stress due to fuselage bending = 80 Mpa for downbend, -80 MPa for upbend
- Shear stress due to torsion of fuselage = +35 Mpa, -35MPa for Case 1 and 2.



# Limit and Ultimate Loads

Exercise-3V

- For general aircraft design which load criterion drives the sizing of the details, limit or ultimate? Study few materials (AL 2024, 7010) and fastener material (Titanium, Steel).
  - Allowable Stresses under Limit load are Yield Strength
  - Allowable Stresses under Ultimate load are Ultimate Strength

# Material Failure Theories

Exercise – 3W

- Fuselage skin sheet is made of 2024 which has a Fty=40ksi. Find the yield RF /MoS for the following conditions.
- Case 1: Fuselage bending produces a tensile stress of 37ksi
- Case 2: Case 1 + hoop stress = 8.6ksi, additional longitudinal stress = 4.3ksi

$$\sigma_{eq} = \sqrt{\sigma_1^{\;2} + \sigma_2^{\;2} - \sigma_1 \cdot \sigma_2} \qquad \sigma_{eq} = \sqrt{\sigma_x^{\;2} + \sigma_y^{\;2} - \sigma_x \cdot \sigma_y}$$

- Case 3: case 2 + shear stress of 8 ksi

$$\sigma_{eq} = \sqrt{\sigma_x^{\;2} + \sigma_y^{\;2} - \sigma_x \cdot \sigma_y + 3 \cdot \tau_{xy}^{\;2}}$$

# Interaction of failure modes

Exercise – 3X

- Calculate the RF and MOS using the different interaction equations for the following condition.

$\sigma_{1\_a} := 350\ \text{MPa}$ $\sigma_{1\_u} := 500\ \text{MPa}$

$\sigma_{2\_a} := 300\ \text{MPa}$ $\sigma_{2\_u} := 480\ \text{MPa}$

$$R_{comb} = R_1 + R_2$$

$$R_{comb}^{\ 2} = R_1^{\ 2} + R_2^{\ 2}$$

# Composite Materials

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- Calculate the load split between fibre and matrix for the following properties

$$A_f := 6\ \text{mm}^2 \qquad E_f := 135000\ \text{MPa} \qquad P := 100\ \text{N}$$

$$A_m := 4\ \text{mm}^2 \qquad E_m := 35400\ \text{MPa}$$

# Sandwich Components

Exercise – 3Z

Calculate the stresses in the facing sheet, core and the deflection due to flexure and shear for the following simply supported beam

Facing sheet thickness $t_f := 0.5\ \text{mm}$

Core thickness $t_c := 25.4\ \text{mm}$

$h := t_c + t_f = 25.9\ \text{mm}$

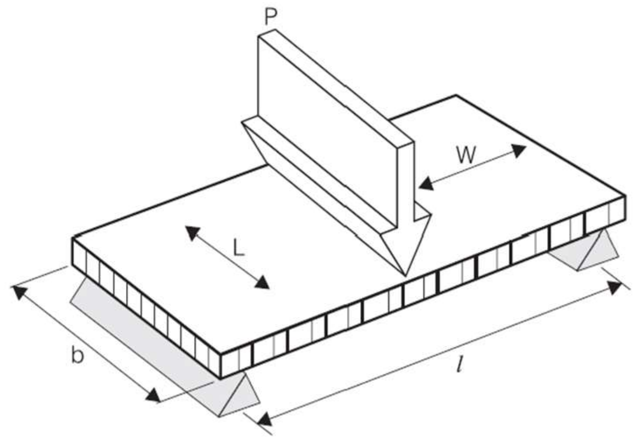
Applied Load $P := 2000\ \text{N}$

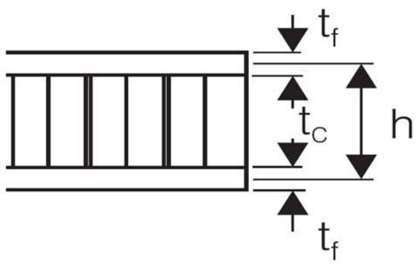
$b := 500\ \text{mm}$

$l := 2000\ \text{mm}$

Modulus of facing sheet $E_f := 70000\ \text{MPa}$

Shear Modulus of core $G_c := 220\ \text{MPa}$



# Exercises

# Module 4: Instability Analysis

# Elastic Buckling of Columns

Exercise – 4A

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- Using Euler equation, calculate the failure stress for a column under the following conditions. Assume pinned end conditions.
- Square cross section with dimension 20 mm, Length of 400 mm, E=70000MPa, c=1

- Calculate at what length the compressive failure mode is more critical than the buckling failure mode if the allowable compressive stress is 450MPa.

# In-Elastic Buckling of Columns

Exercise – 4B

- Calculate the critical buckling stress for the lengths of 800 mm and 600 mm for the following member.

$E := 70000\ \text{MPa}$

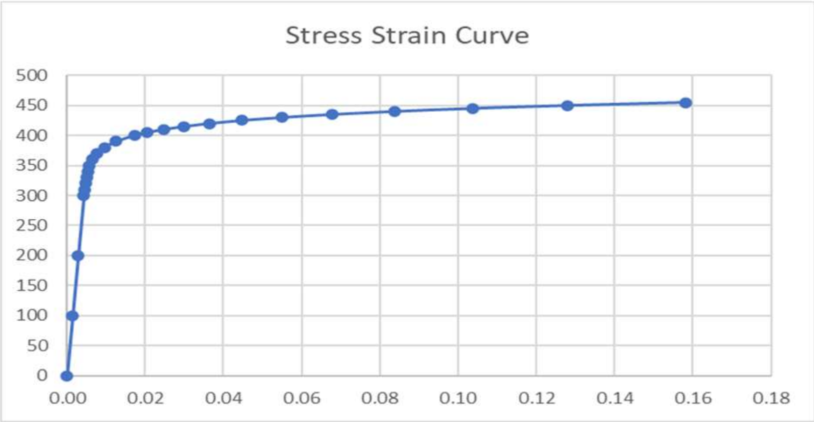
$L_1 := 800\ \text{mm}$

$c := 1$

$I := 53913\ \text{mm}^4$

$A := 240\ \text{mm}^2$

| | | |
|---|---|---|
| E | 70000 | Mpa |
| K | 500 | MPa |
| n | 20 | |



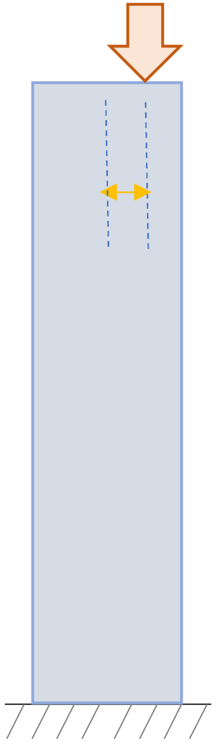
# Buckling of Columns – Eccentric Loading

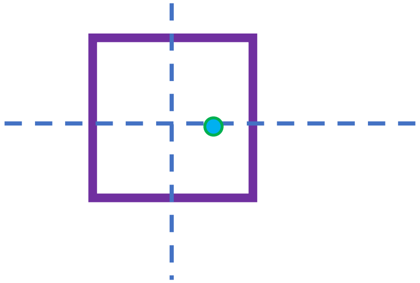
Exercise – 4C

- Calculate the maximum compressive stress for the following column under the eccentric loading.

$b := 20\ \text{mm}$ $P := 40000\ \text{N}$ $E := 70000\ \text{MPa}$

$L := 400\ \text{mm}$ $e := 5\ \text{mm}$





Calculate the maximum allowable load if the yielding is not to occur Fty=380 MPa

# Buckling of Columns

Exercise – 4D

- Calculate the Reserve Factor for the Cessna 172 strut made of steel if a circular cross section is chosen of Outer Diameter 50 mm and inner diameter 38mm. Assume Pinned end constraints.
- *Case 1: Maximum tensile load = 50344 N*
- Case 2: Compressive load = 22880 N

$P_1 := 50344\ \mathrm{N} \quad P_2 := -22880\ \mathrm{N}$

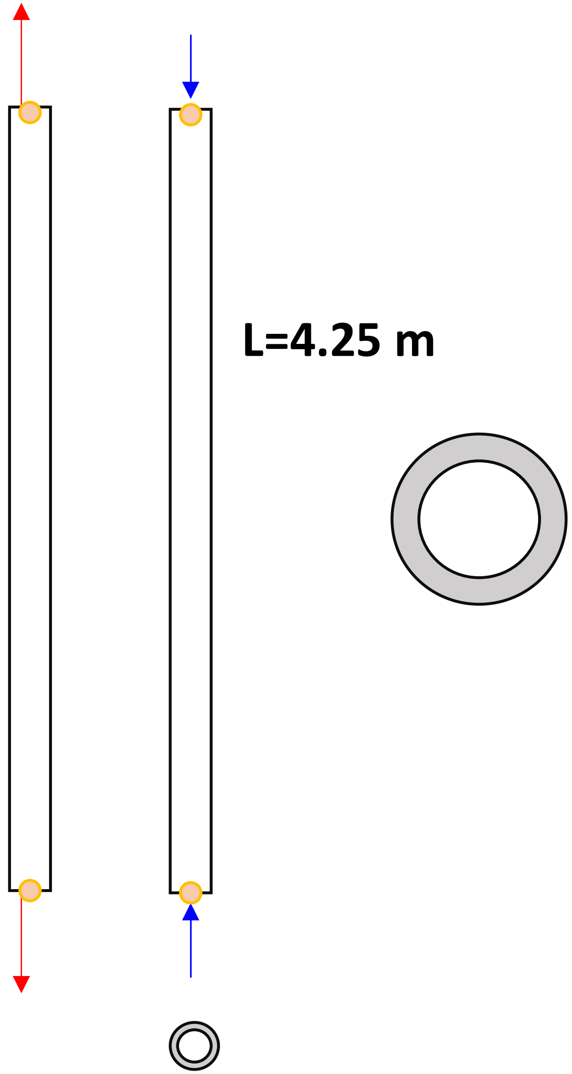
$D_o := 50\ \mathrm{mm} \quad D_i := 38\ \mathrm{mm}$

$E := 210000\ \mathrm{MPa}$

$c := 1$ *for Pinned Ends*

$\sigma_{tu} := 1200\ \mathrm{MPa}$

$L := 4.25\ \mathrm{m}$

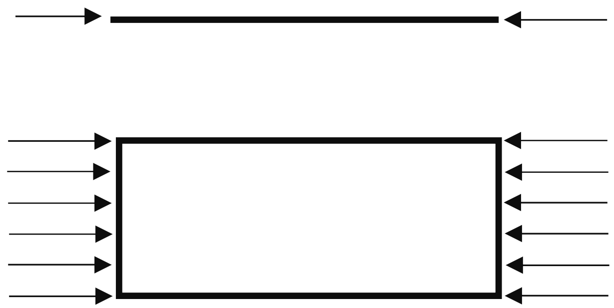



Can the margin be improved without increasing the weight by considering the loading action and changing the cross section shape ?

# Buckling of Panels

Exercise – 4E

- Calculate the allowable compressive stress & load for the following panel.



$E := 70000\ \text{MPa}$  $b_1 := 200\ \text{mm}$

$\upsilon := 0.33$  $b_2 := 800\ \text{mm}$

$t := 2\ \text{mm}$

# Buckling of Panels - Stiffened

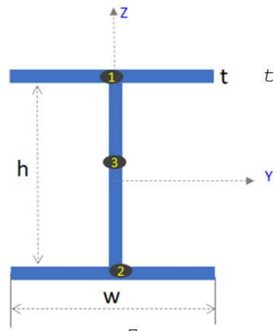
- Calculate the critical buckling load for the following Stiffened Panel (Panel + Stiffeners).

$t = 2\ \mathrm{mm}$

$P_{cr} = 10337\ \mathrm{N}$

$t := 1\ \mathrm{mm}$

$h := 11\ \mathrm{mm}$




$t := 4\ \mathrm{mm}$

$w := 7\ \mathrm{mm}$

$E := 70000\ \mathrm{MPa}$ $\quad b_1 := 200\ \mathrm{mm}$

$\upsilon := 0.33$ $\quad b_2 := 800\ \mathrm{mm}$

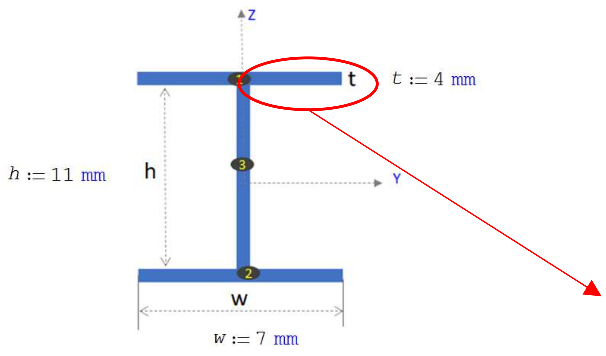
$t := 1\ \mathrm{mm}$ $\quad k_c := 4$

$b := b_1$

# Buckling of stiffeners

Exercise – 4G

- Calculate the critical buckling stress for the stiffener flange




$E := 70000$ MPa

$b_1 := 3.5$ mm

$\upsilon := 0.33$

$b_2 := 800$ mm

$t := 4$ mm

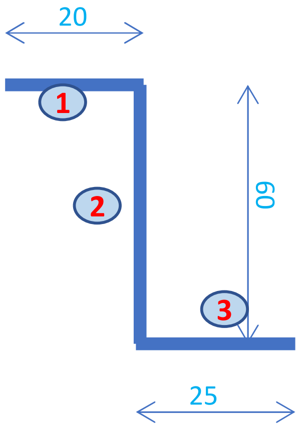
$k_c := 0.43$

$b := b_1$

# Crippling Failure

Exercise – 4H

- Find out the crippling stress for the following cross section, t=0.65 mm, Material 2024-T3 Clad.

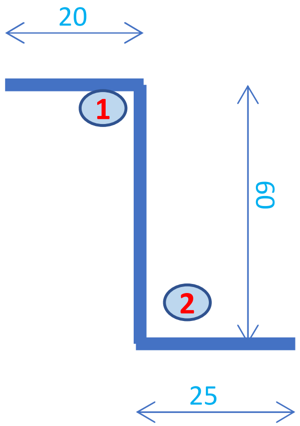



$$F_{cc} = \frac{\sum b_n t_n F_{ccn}}{\sum b_n t_n}$$

# Crippling Failure

Exercise – 4I

- Calculate the crippling allowable using the angle method (Needham Method)



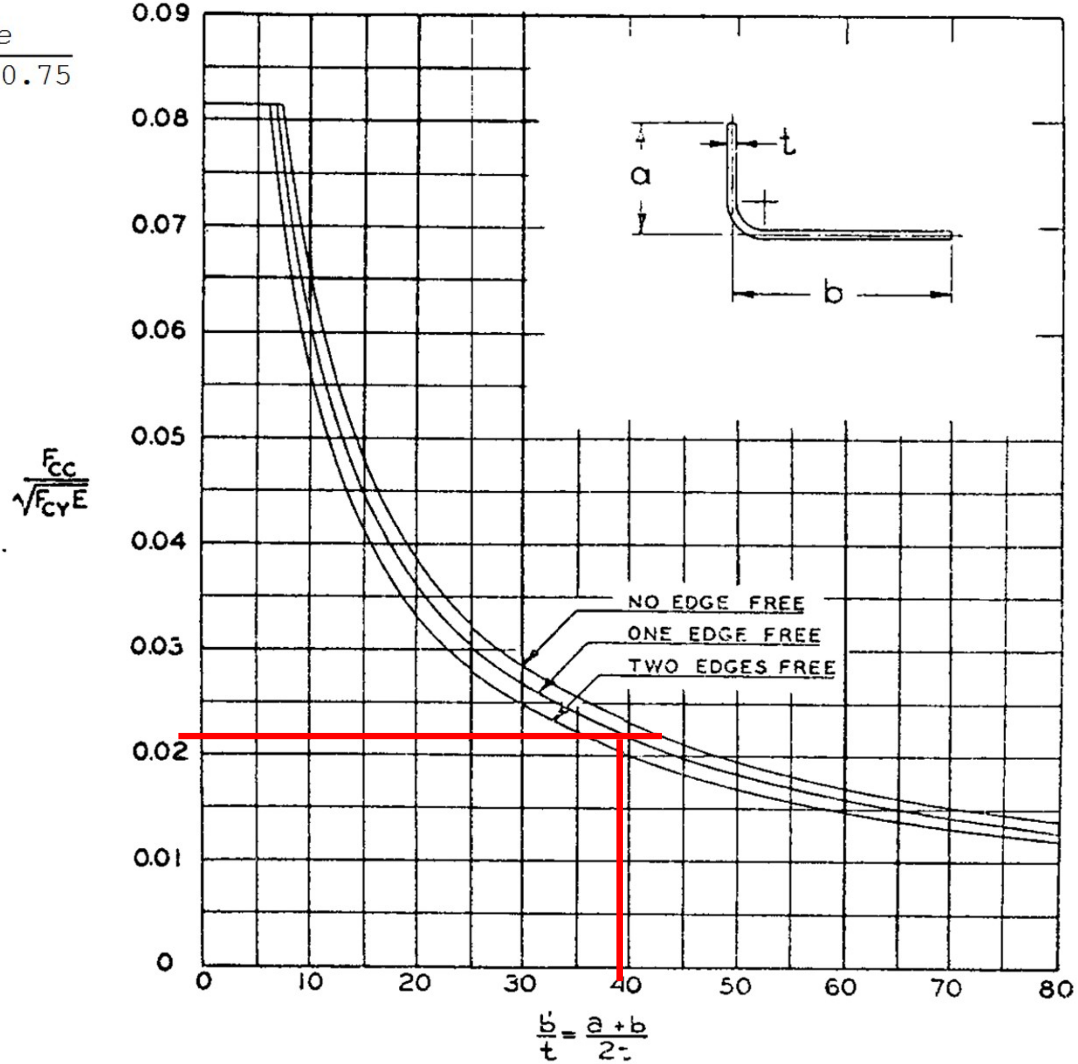

$C_e$ Coefficient of support, 0.316 two edges free, 0.342 one edge free, 0.366 no edges free

$$\sigma_{cy} := 350\ \text{MPa}$$

$$E := 70000\ \text{MPa}$$

$$C_e := 0.342$$

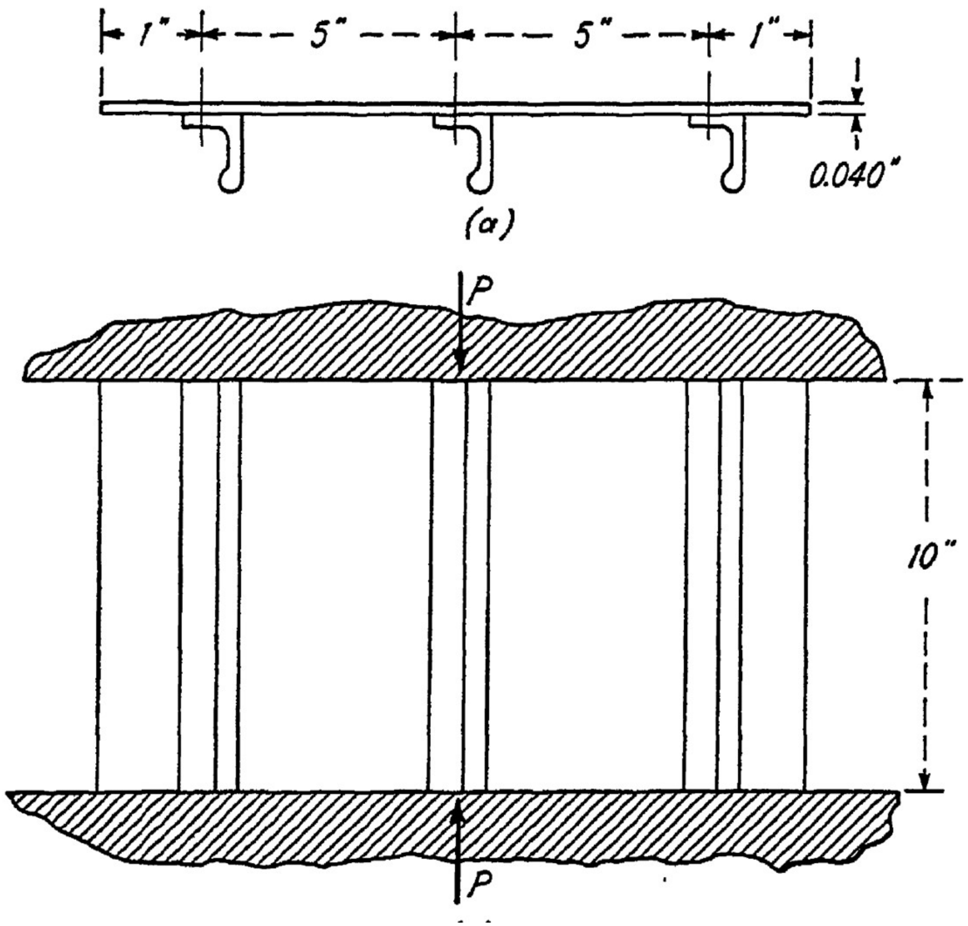
$$\frac{\sigma_{cc}}{\sqrt{\sigma_{cy} \cdot E}} = \frac{Ce}{\left(\frac{b'}{t}\right)^{0.75}}$$



# Effective Widths in Stiffened Panel

Exercise – 4J

- Find out the buckling/crippling load
- Case 1: when the sheet first buckles
- Case 2: when the stringer stress is 10ksi
- Case 3: when the stringer stress is 30 ksi
- Stringer area = 0.1 in^2 ,
- E=10300 ksi for both panel and stringer



## Inter Rivet Buckling

Exercise – 4K

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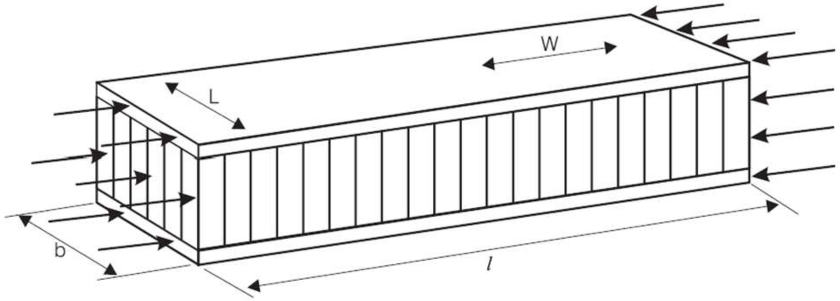
- Find the critical compressive stress that will lead to inter rivet buckling for fuselage skin made of aluminium 2024 (E=70000MPa) when the pitch is 6d and d/t ratio of 1.5. Skin to stringer attachment is by countersunk rivets.

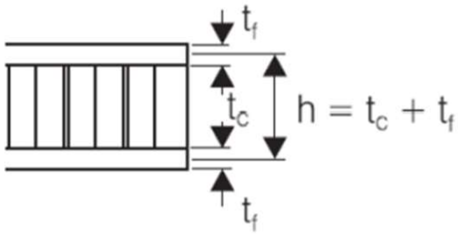
# Buckling of Composite Panels

Exercise – 4L

Calculate the Allowable Stresses for different failure modes of the sandwich panel.

Facing Skins Aluminium 5251 H24, Core 5.2 - 1/4 - 3003




Facing sheet thickness $t_f := 0.5\ mm$

Core thickness $t_c := 25.4\ mm$

$h := t_c + t_f = 25.9\ mm$

$b := 500\ mm$

$l := 2000\ mm$

Modulus of facing sheet $E_f := 70000\ MPa$

Moduli of core $E_c := 1000\ MPa$

$G_c := 220\ MPa$

using min of GL and GW

Size of the cell $s := 6.35\ mm$

# Combined Loading

Exercise – 4M

- Calculate the margin for a panel subjected to compressive stress of 1900psi and shear stress of 900 psi.
- Panel thickness is 0.04" and is 5" by 8".
- E=10700ksi, υ=0.3

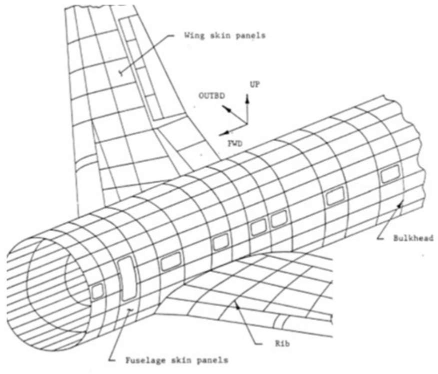
$\sigma_1 := 1900\ \text{psi}$ $\tau_1 := 900\ \text{psi}$

$E := 10700\ \text{ksi}$ $\upsilon := 0.3$

$b := 5\ \text{in}$ $t := 0.04\ \text{in}$

$$R_c + R_s^{\ 2} = 1$$

$$MoS := \frac{2}{R_c + \sqrt{R_c^{\ 2} + 4 \cdot R_s^{\ 2}}} - 1$$



# Exercises

# Module 5: Joints

# Failure Modes

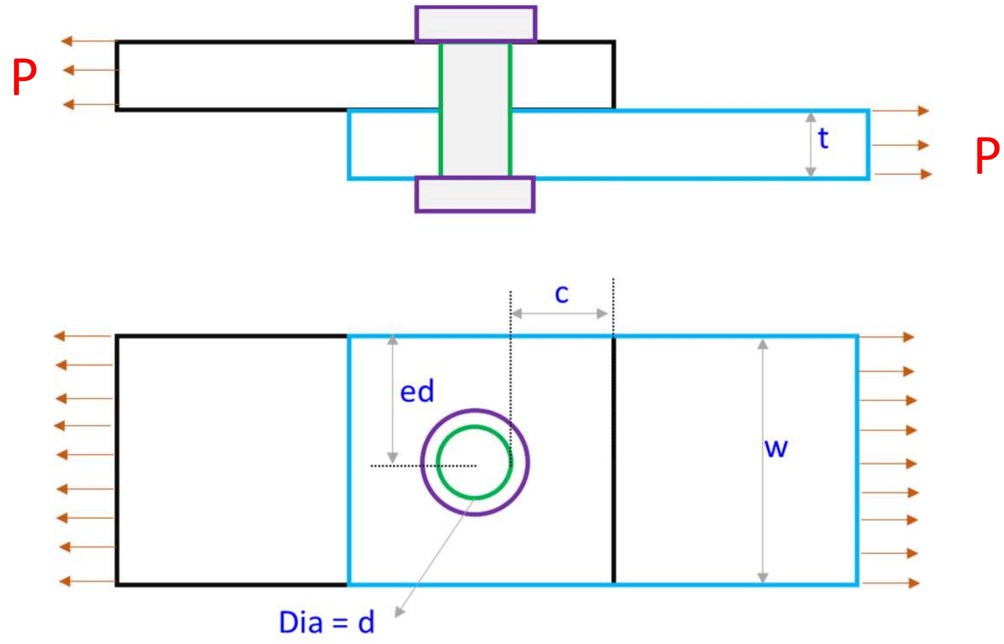
Exercise – 5A

- Plate material:7010-T7451, Fastener: Ti-6Al-4V Bar
- Applied Ultimate load of 60 kN
- Calculate the RF's for different failure modes of Fastener and plate.

$P := 60000\ N$

$t := 10\ mm$ $w := 40\ mm$

$d := 12.5\ mm$ $c := 20\ mm$



# Shear Strength of Rivets

Exercise – 5B

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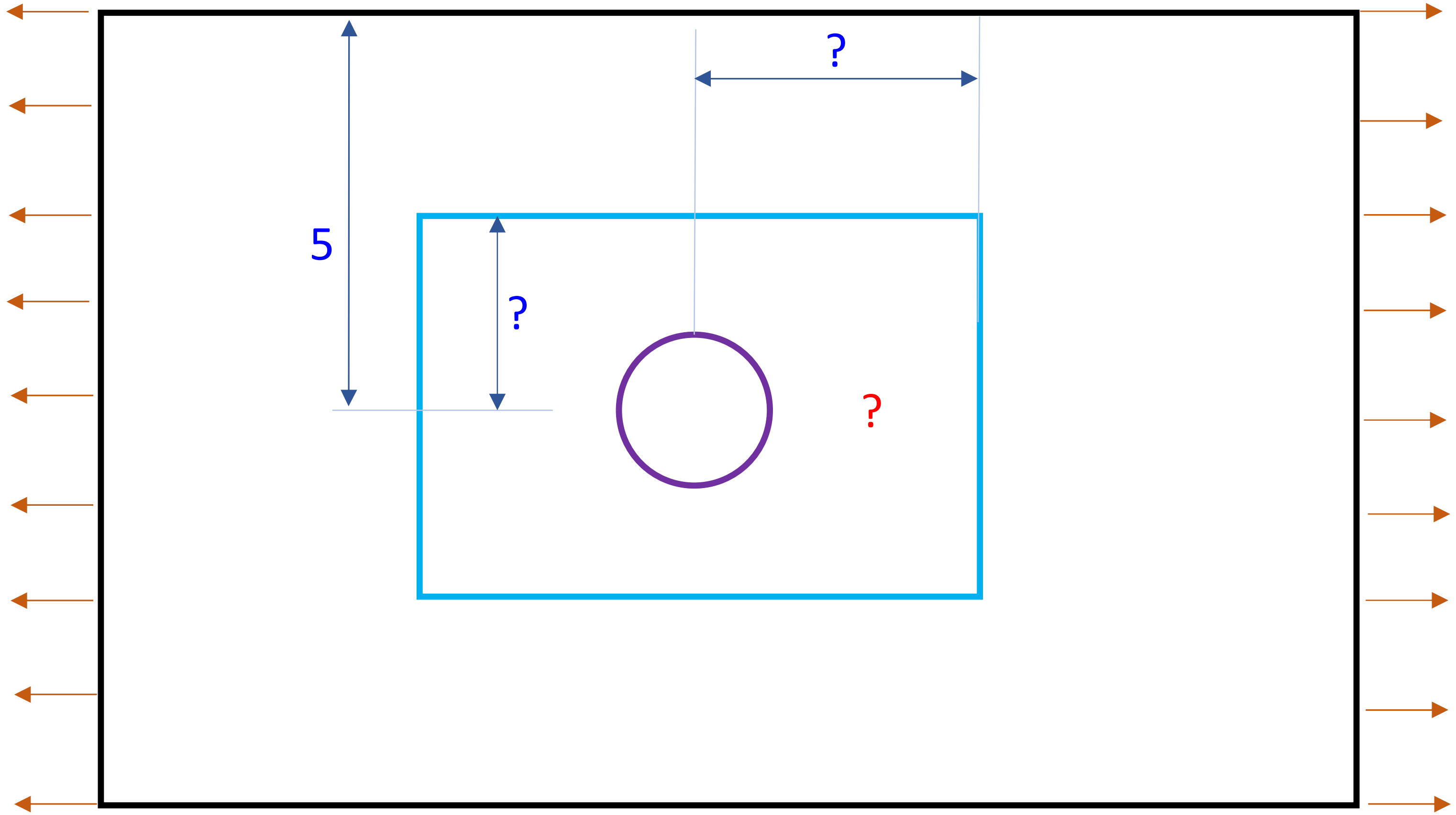
- Calculate the allowable shear load on MS14218AD rivet of 3/16” dia with $\sigma_{su}$ of 30ksi

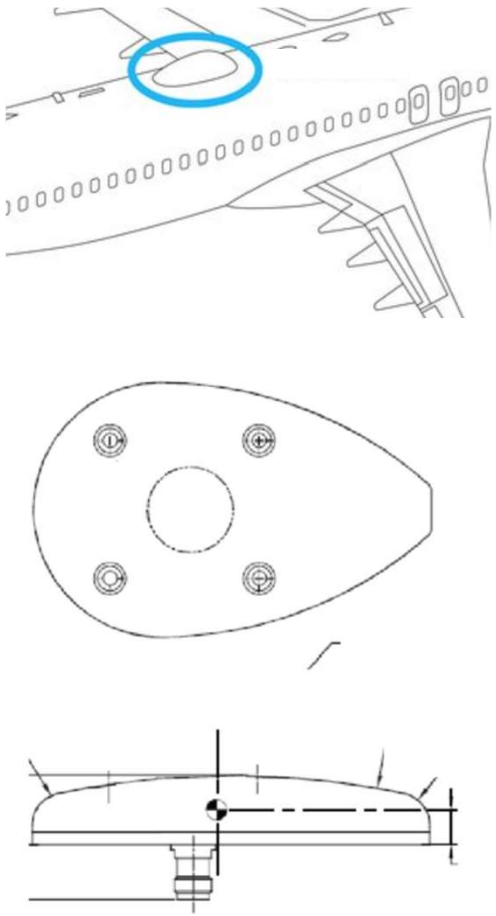
| Rivet Type ................ | MS14218AD[a] ($F_{su}$ = 30 ksi) | | | | | |
|---|---|---|---|---|---|---|
| Sheet Material ............. | Clad 2024-T3 | | | | | |
| Rivet Diameter, in. .......... (Nominal Hole Diameter, in.)[b] . | 3/32 (0.096) | 1/8 (0.1285) | 5/32 (0.159) | 3/16 (0.191) | 7/32 (0.228) | 1/4 (0.257) |

# Riveted Joint Design

Exercise – 5C

- Antenna installation requires drilling a cutout of 2in in a skin panel (2024-T3) of width 10in, thickness 0.04in
- Design a doubler joint as shown to the skin of a fuselage by "**returning to original strength**" approach, doubler thickness same as skin.
- Use protruding rivet "AD"

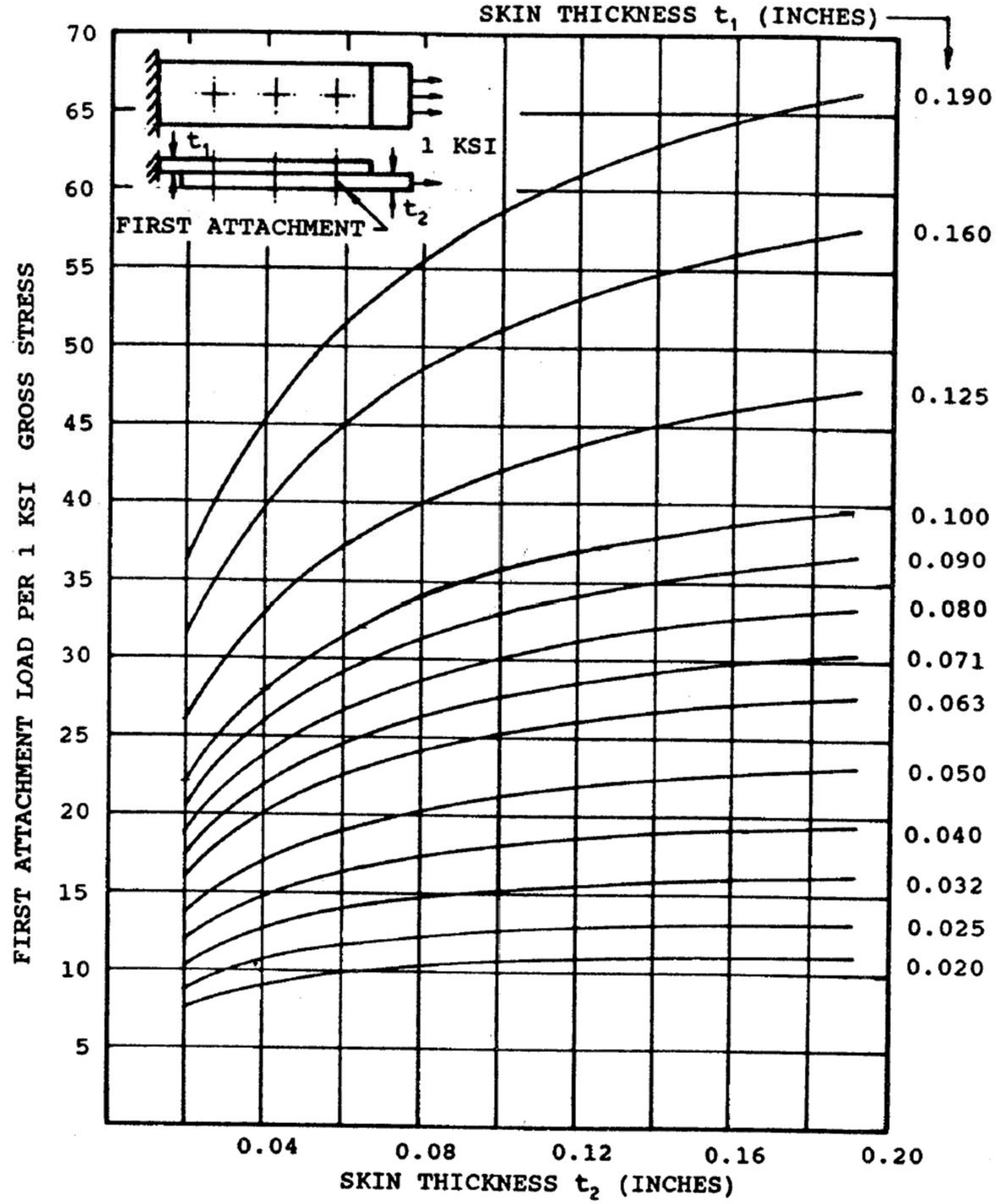


# Load distribution in Lap and Butt Joints

Exercise – 5D

- Calculate the peaking factor for skin and doubler thickness combinations of
- a) 0.04in  b) 0.16in



# Bolt Group Analysis

Exercise – 5E

- Calculate the shear load on the fasteners for the joint shown.

- Load application point x = 2 in, y = -2 in

$P_x := 800\ \text{lbf} \qquad P_y := -300\ \text{lbf}$

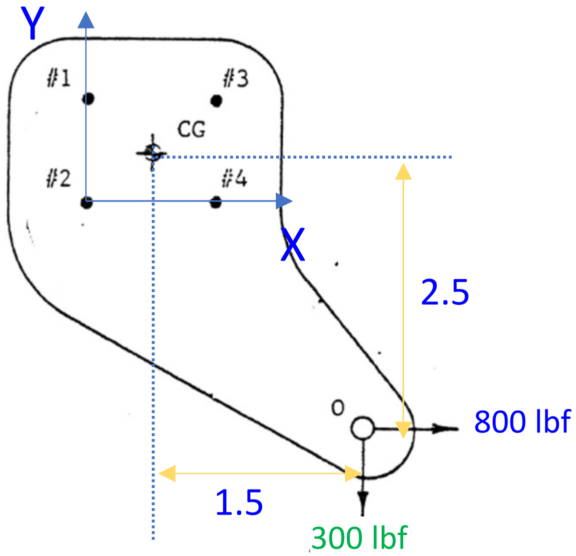
$n := 4$

$$p_{x1} := \frac{P_x}{n} = 200\ \text{lbf}$$

$$p_{y1} := \frac{P_y}{n} = -75\ \text{lbf}$$

$x' := 1.5\ \text{in} \qquad y' := -2.5\ \text{in}$

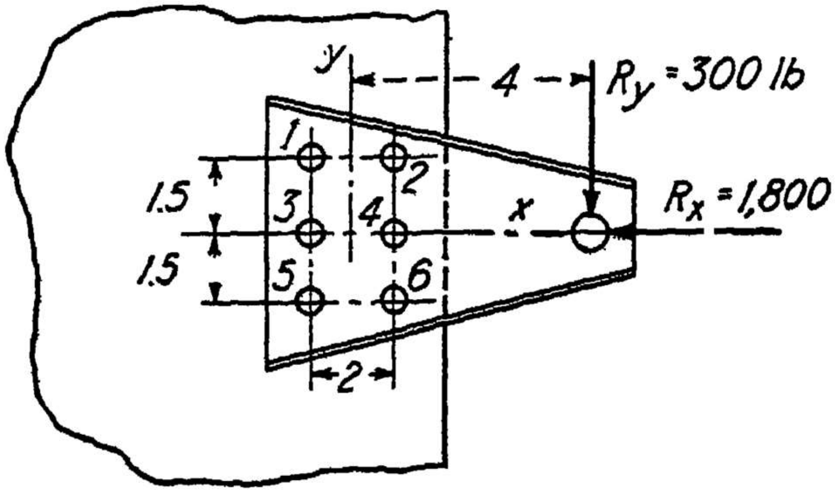
| Fastener | x | y |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 0 | 0 |
| 3 | 1 | 1 |
| 4 | 1 | 0 |



# Bolt Group Analysis

Exercise – 5F

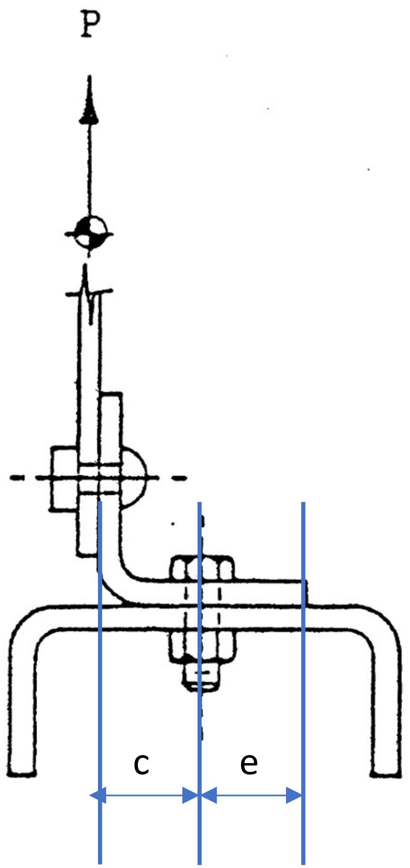
- Calculate the shear load on the rivets for the joint shown.

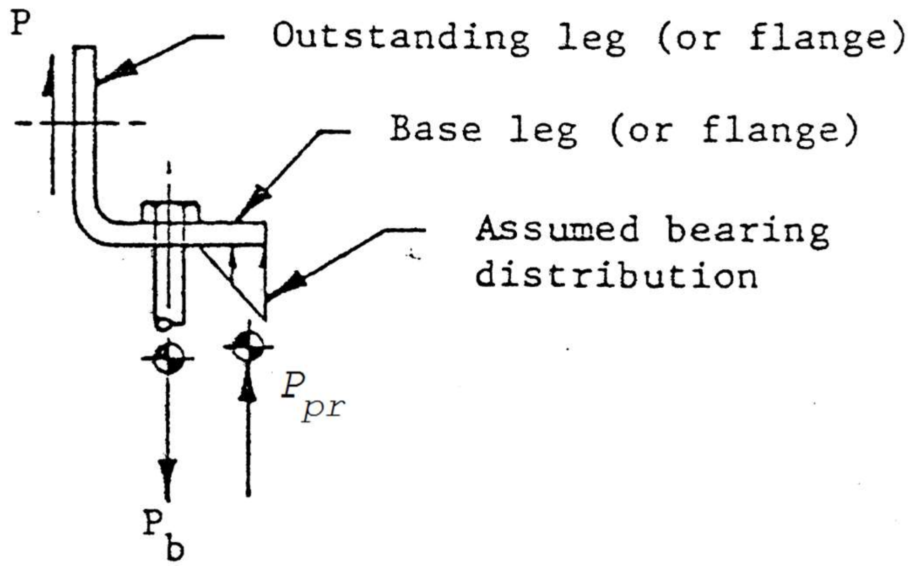


# Tension Clip

Exercise – 5G

- Write down the equation for tension load in the bolt for the following tension clip

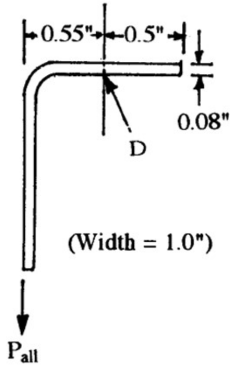


# Strength of Tension Clips

Exercise – 5H1

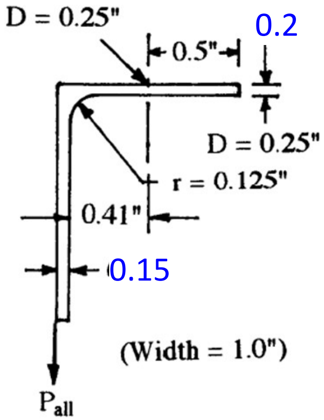
- Calculate the allowable load for the following tension clip (7075)



# Strength of Tension Clips

Exercise – 5H2

- Calculate the allowable load for the following tension clip



## Strength of Tension Clips

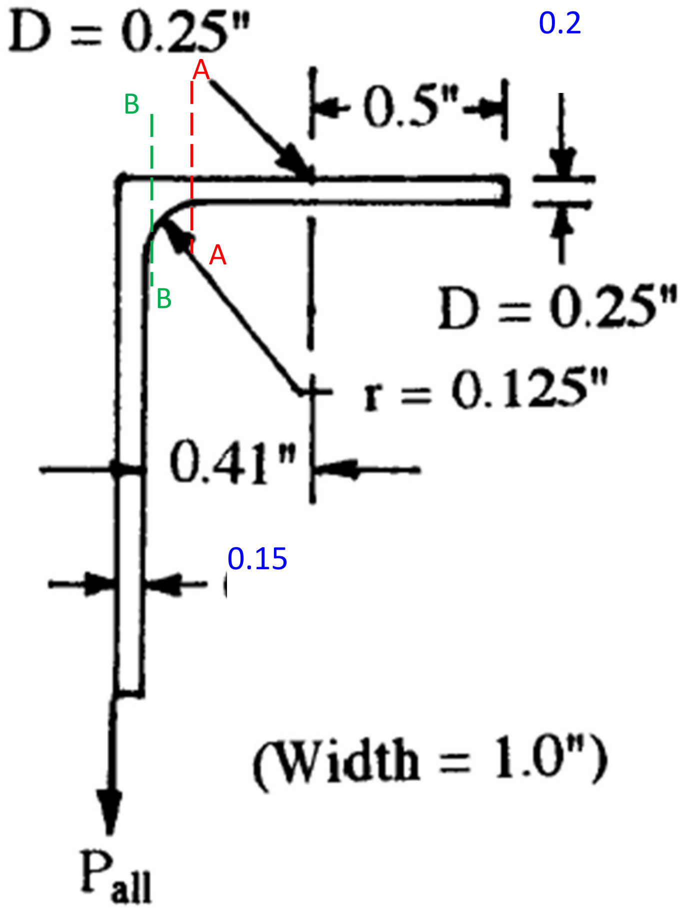
Exercise – 5H2

- Calculate the bending stress at the fastener attachment location for the maximum allowable load
- Consider a width of 5d

$P := 2800\ lbf$

$c := 0.41\ in$

$e := 0.5\ in$




$P_{all}$ = 2800 lbf

# Interaction of Shear & Tension - Bolts

Exercise – 5I

- Calculate the RF for a 5/8 in AN Steel bolt which is subjected to following loads.
- Shear = 19000 lbf, Tension = 7000 lbf

$P_s := 19000\ \text{lbf}$ $\quad P_t := 7000\ \text{lbf}$

$P_{su} := 23000\ \text{lbf}$ $\quad P_{tu} := 30100\ \text{lbf}$

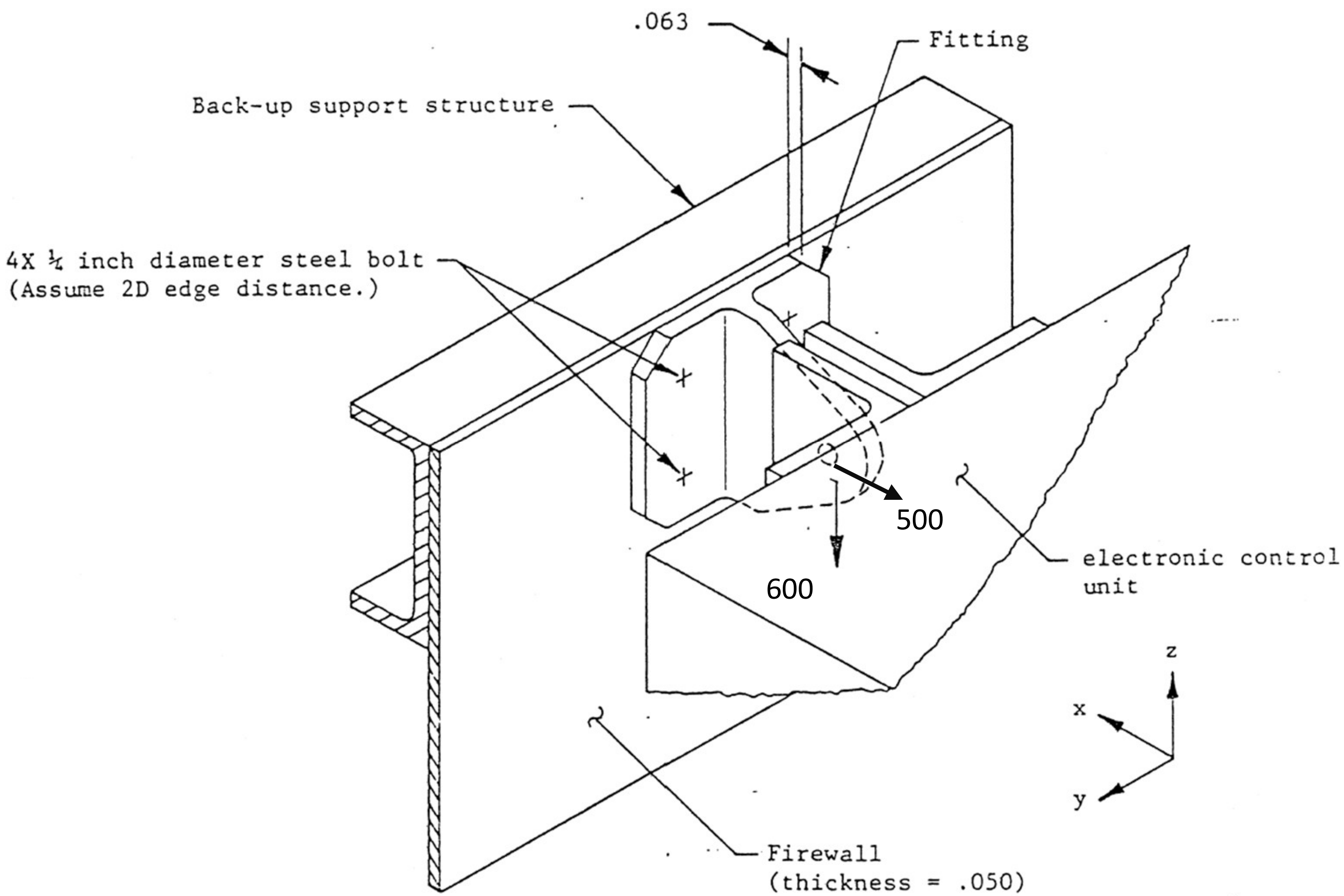
Find out RF such that

$$\left(RF \cdot R_s\right)^3 + \left(RF \cdot R_t\right)^2 = 1$$

# Fittings – Shear and Tension Interaction

Exercise – 5J

- Calculate the RF for the fastener AN-5 (shear tension interaction) and maximum stress in the T clip shown below for Fx = -500 lbf, Fz = -600 lbf
- Bracket width 2 in by 2 in, Offset in x = 2 in



# Lugs – Axial Loaded

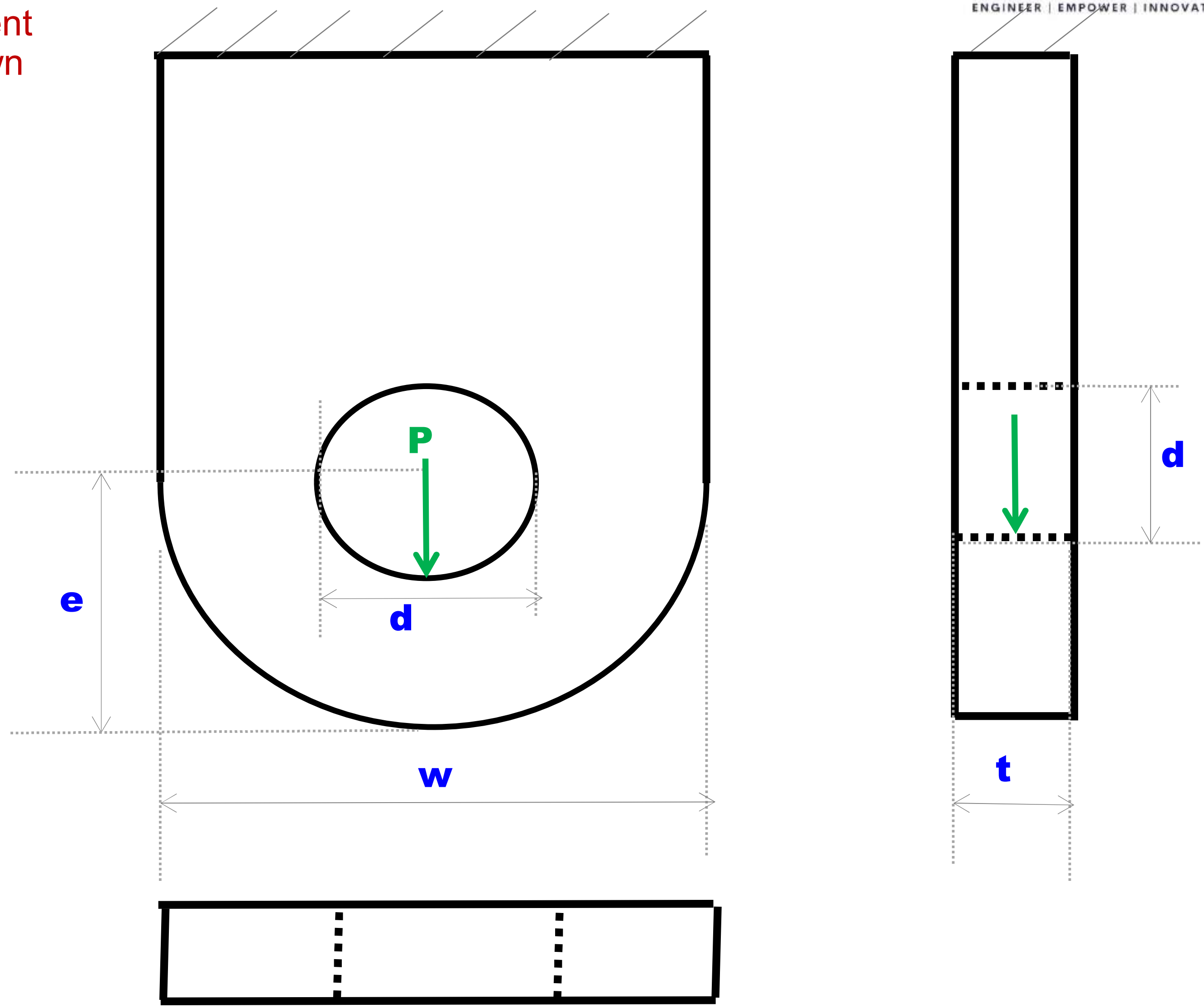
Exercise – 5K

- Calculate the Reserve Factors for different failure modes for the lug (2024-T4) shown
- Shear-out and Bearing Failure
- Net section failure

$d := 30\ \text{mm}$ $t := 4\ \text{mm}$ $w := 90\ \text{mm}$

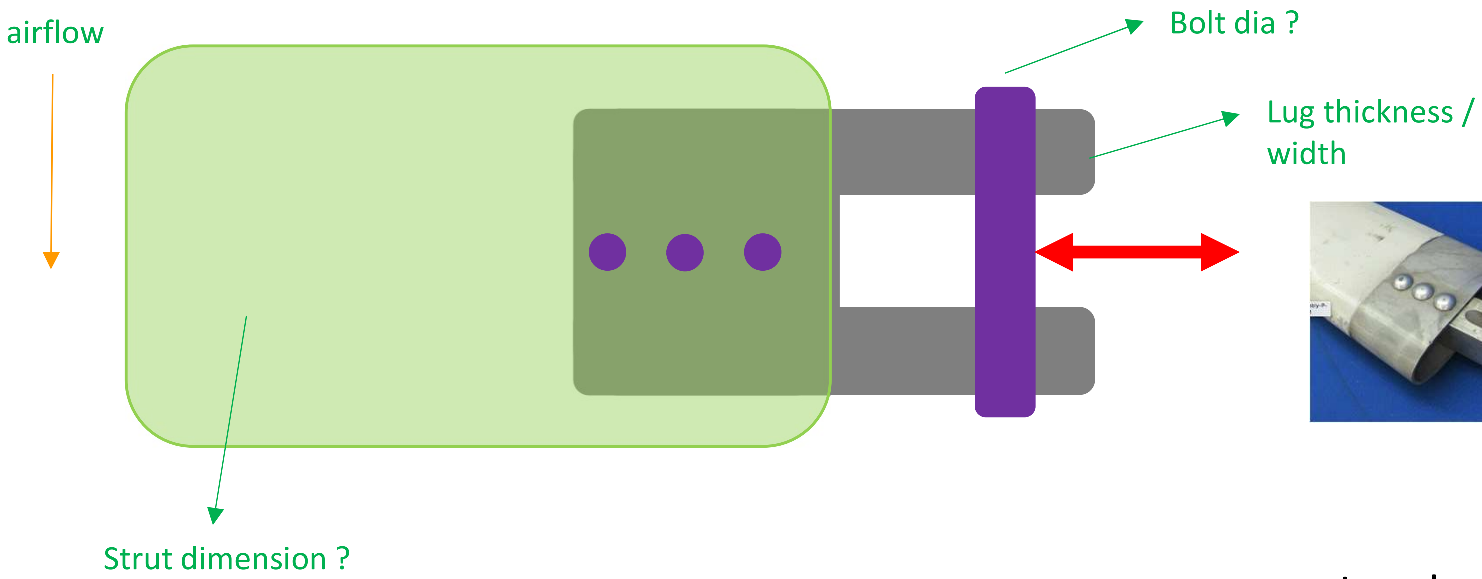
$P := 40000\ \text{N}$ $e := 30\ \text{mm}$

$\frac{e}{d} = 1$ $\frac{w}{d} = 3$ $\frac{d}{t} = 7.5$

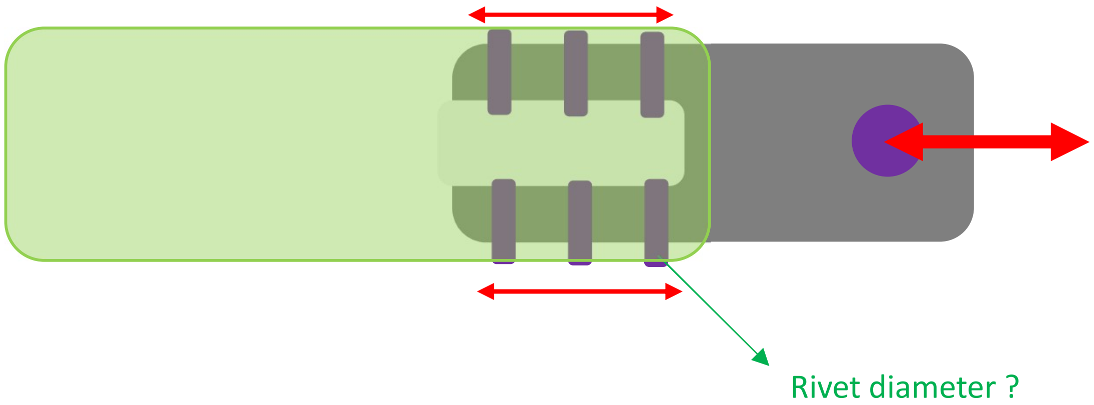


# Lug and Rivets

- Design the Cessna Strut lug and rivet attachment for the given loading



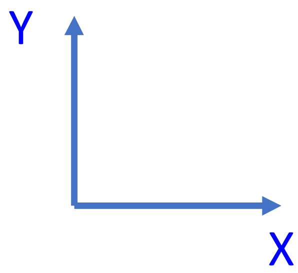

Load on the bolt = 50344 / -22880

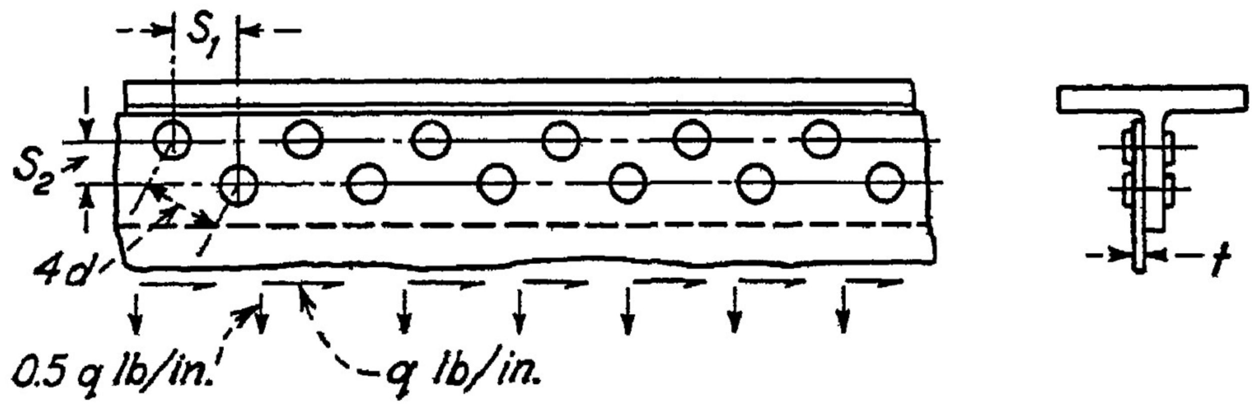


# Loads on Joint due to Shear

Exercise – 5M

- Skin to Spar / Rib to Skin / Rib to Spar / Fuselage skin joint
- Design the joint with shear flow q=1400 lb/in (as shown in figure) and web is Al Clad 2024T3 with t=0.04 in.
- Choose d/t to have shear strength of rivet and bearing strength approx. equal.
- Minimum rivet spacing of 4d



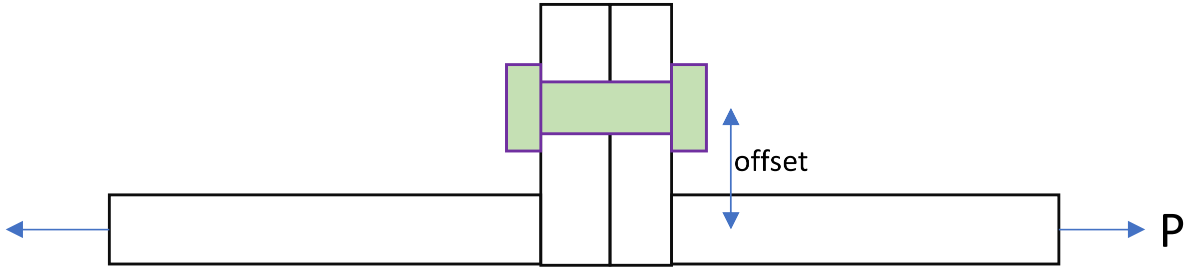




- *Bearing ultimate of a material is approx. 3 times of shear ultimate.*

# Pre Torque

Exercise – 5N

- What should be the pre torque on the fasteners of the joint if the opening of the plates is to avoided throughout service.
- Effective width for bending at fastener = 4d
- Assume the joint is designed with RF=1 for yield.




$offset := 20\ \text{mm}$

$t := 5\ \text{mm}$

$d := 9\ \text{mm}$

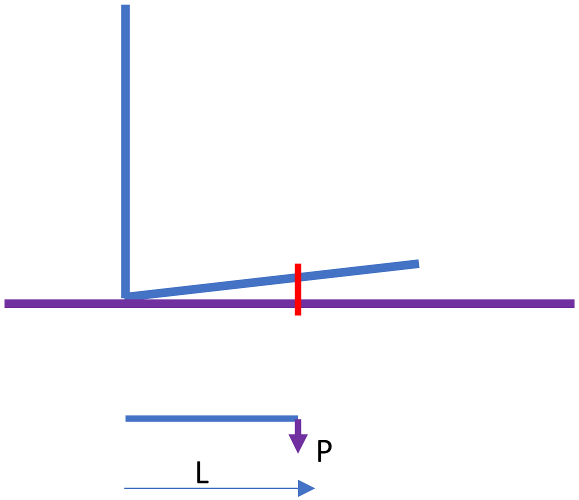
$w := 4 \cdot d = 36\ \text{mm}$

$\sigma_{ty} := 350\ \text{MPa}$

## Shimming and build stresses

Exercise – 50

- Calculate the build stress developed for the following angle if the gap at the bolt location is 0.5mm when assembled.
- 2024T3 sheet thickness of 2 mm, attached using a bolt d/t=4 and edge distance of 2d to free edge and web.




$t := 2\ mm$

$d := 4 \cdot t = 8\ mm$

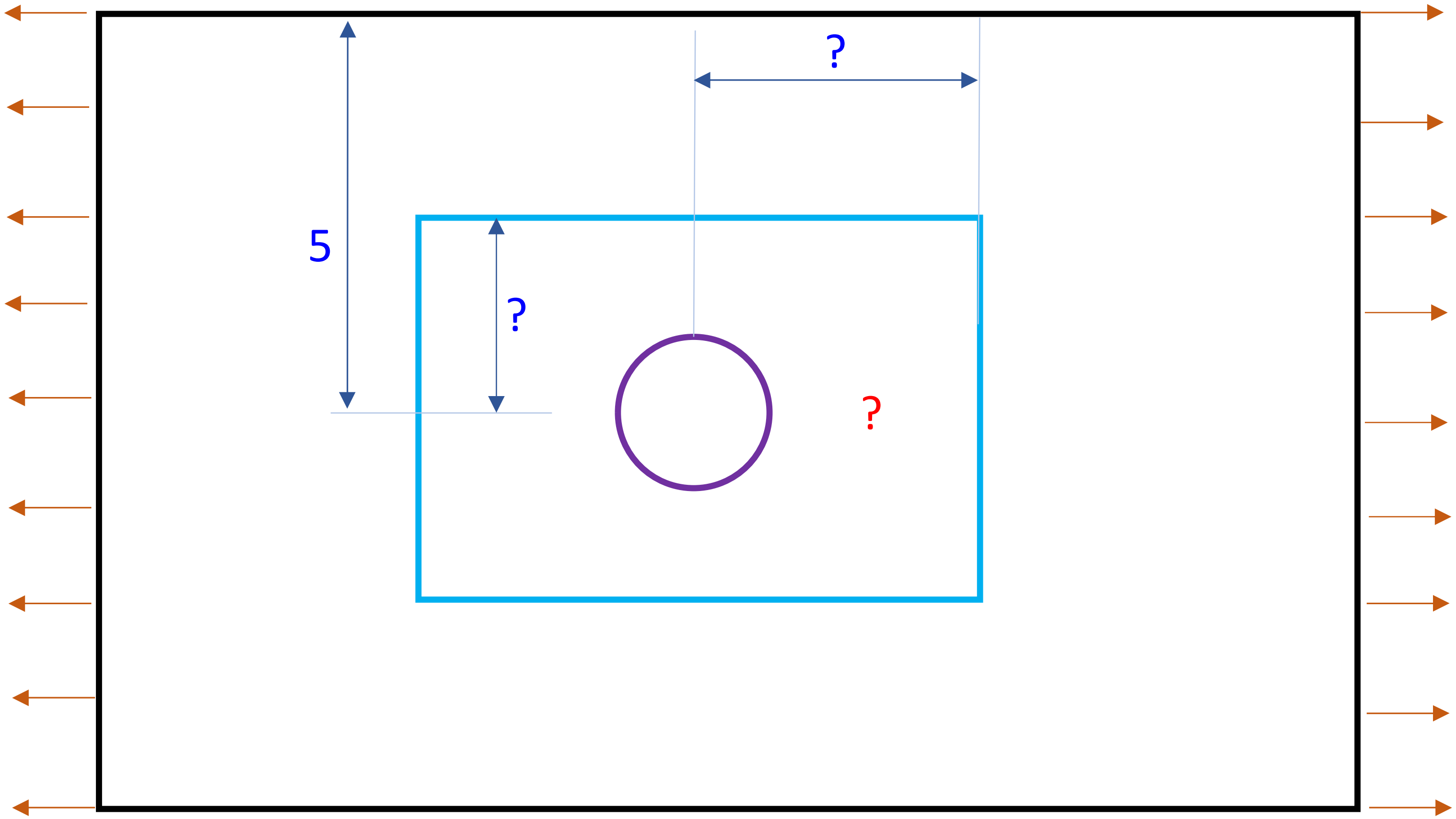
$e := 2 \cdot d = 16\ mm$

$E := 70000\ MPa$

# Bonded Joints

Exercise – 5P

- Design the adhesive joint for the following repair. Base material is 2024
- Adhesive used is LOCTITE EA 9466 with lap shear strength of 2.5 ksi



# Finite Element Analysis – Joint Stiffness

Exercise – 5Q

- Calculate stiffness of a riveted joint of two plates of thickness 1mm and rivet dia of 4mm.
- SWIFT Formula
- E = 70000MPa

$A := 5$

$B := 0.8$

$d := 4\ \text{mm}$

$t_1 := 1\ \text{mm}$

$t_2 := 1\ \text{mm}$

$E := 70000\ \text{MPa}$

$$f := \frac{\left(A + B \cdot \left(\frac{d}{t_1} + \frac{d}{t_2}\right)\right)}{E \cdot d}$$

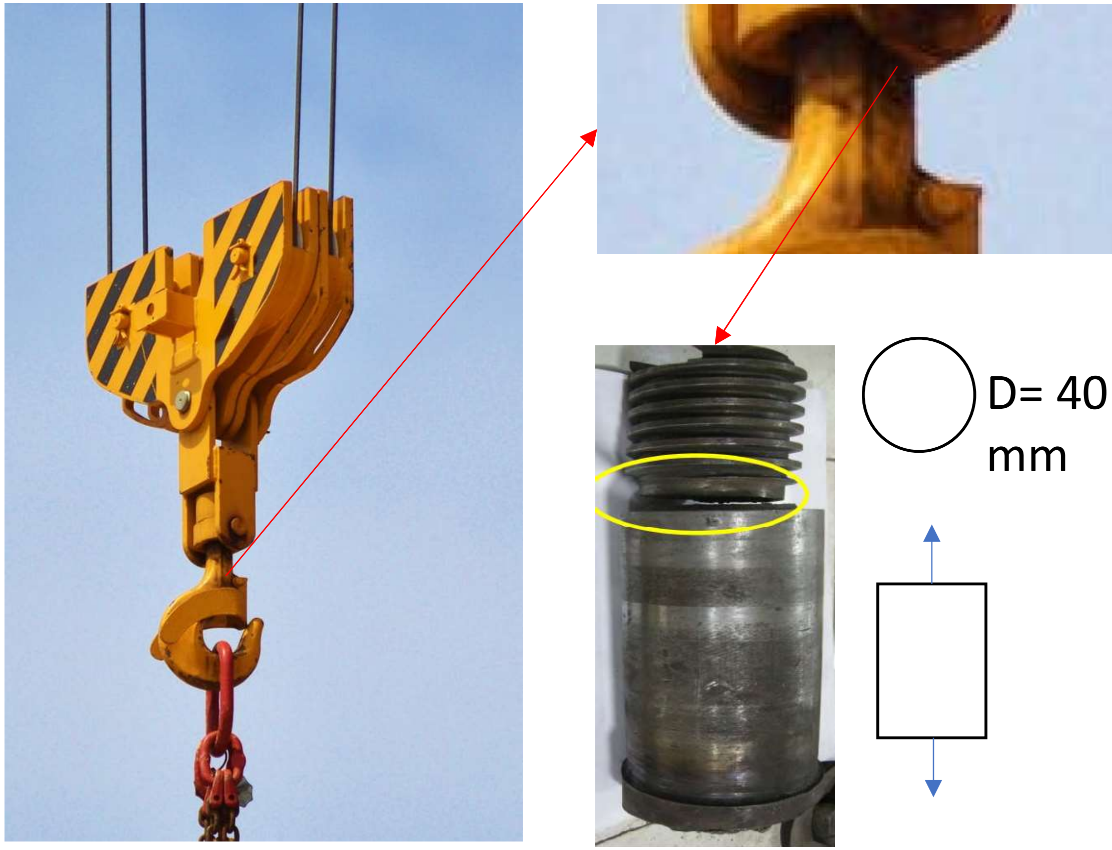
# Exercises

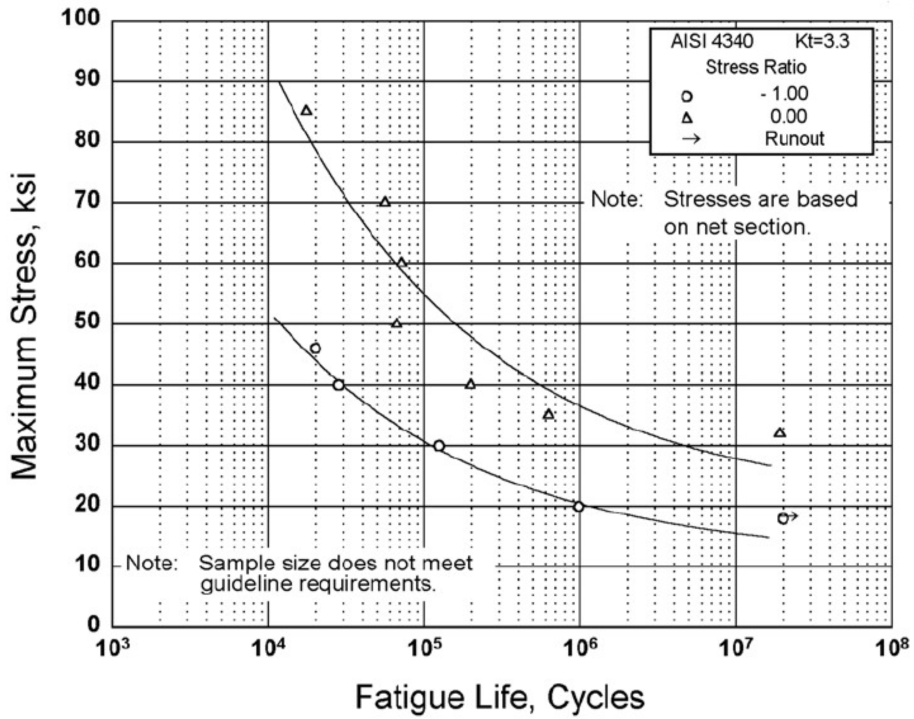
# Module 6: Fatigue and Damage Tolerance

# Fatigue

Exercise – 6A

- Calculate the fatigue life (number of loading cycles) of the hook (Material: AISI 4340 and Diameter of 40mm) for a repeated loading of 0 – 40 Tons. Use a Kt of 3 and the shown S-N Curve.




**Life can be read from the plot as ~ 450,000 Cycles**

# Fatigue

Exercise – 6B

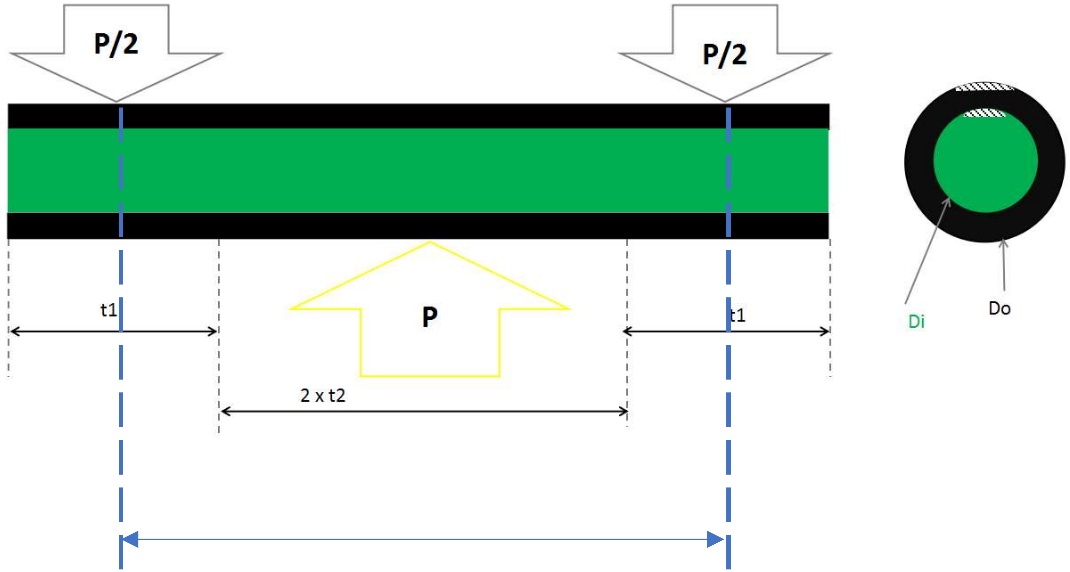
- Calculate the fatigue life of the pin:
- t1 = t2 = 40 mm
- Do = 55 mm, Di = 45 mm
- Load Cycle is Pmax=150 kN, Pmin = 0 kN
- S-N curve is represented by the following equation

- A = 1 x $10^{15}$
- b = -5

Fatigue Life $N := A \cdot \sigma_a^{\,b}$

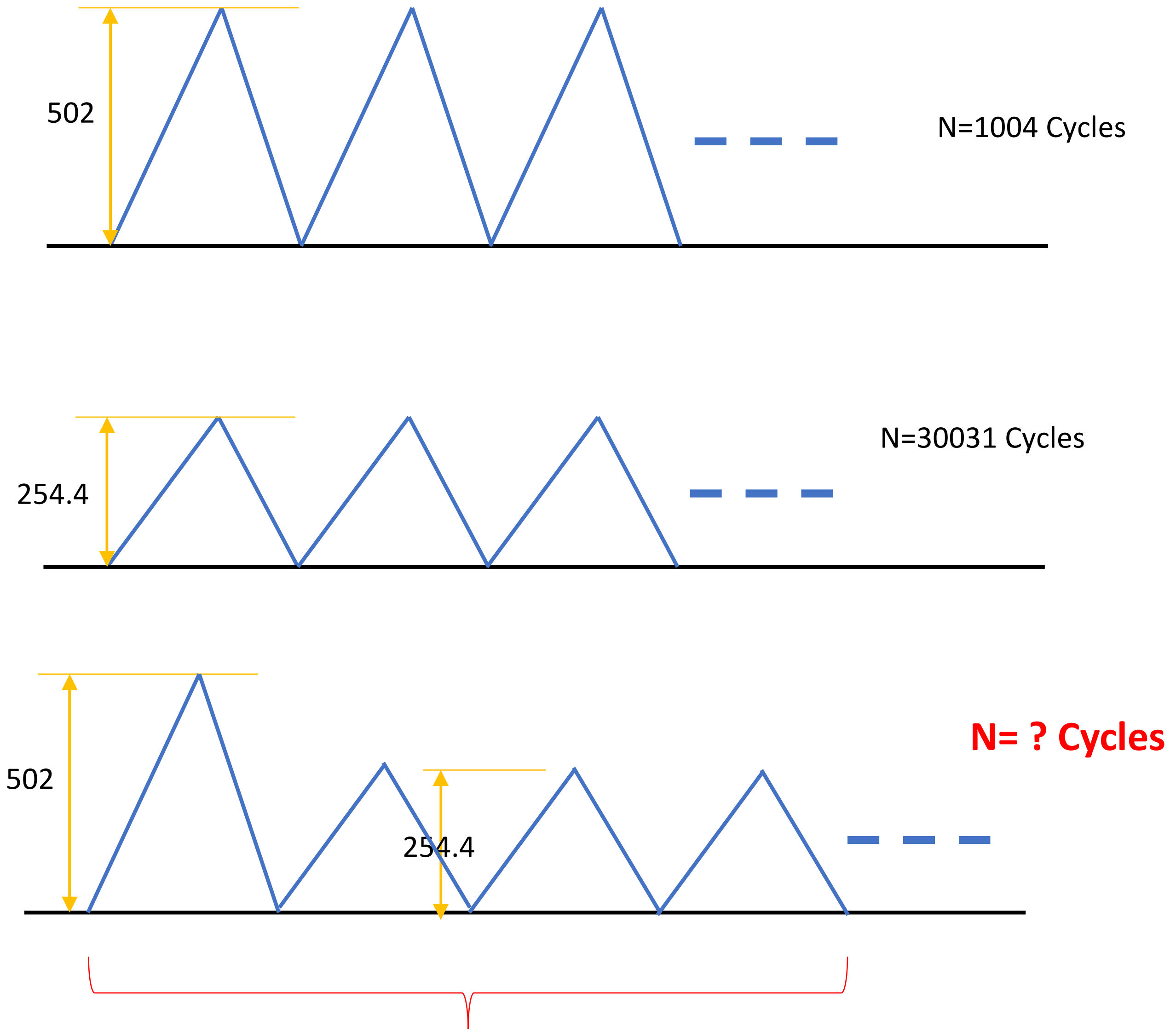
$\sigma_a$ Alternating Stress

A , b are Material Constants



# Concept of accumulated damage

Exercise – 6C



- Effective stress ratio for cyclic loading with mean stress

$A := 5.82 \cdot 10^{13}$

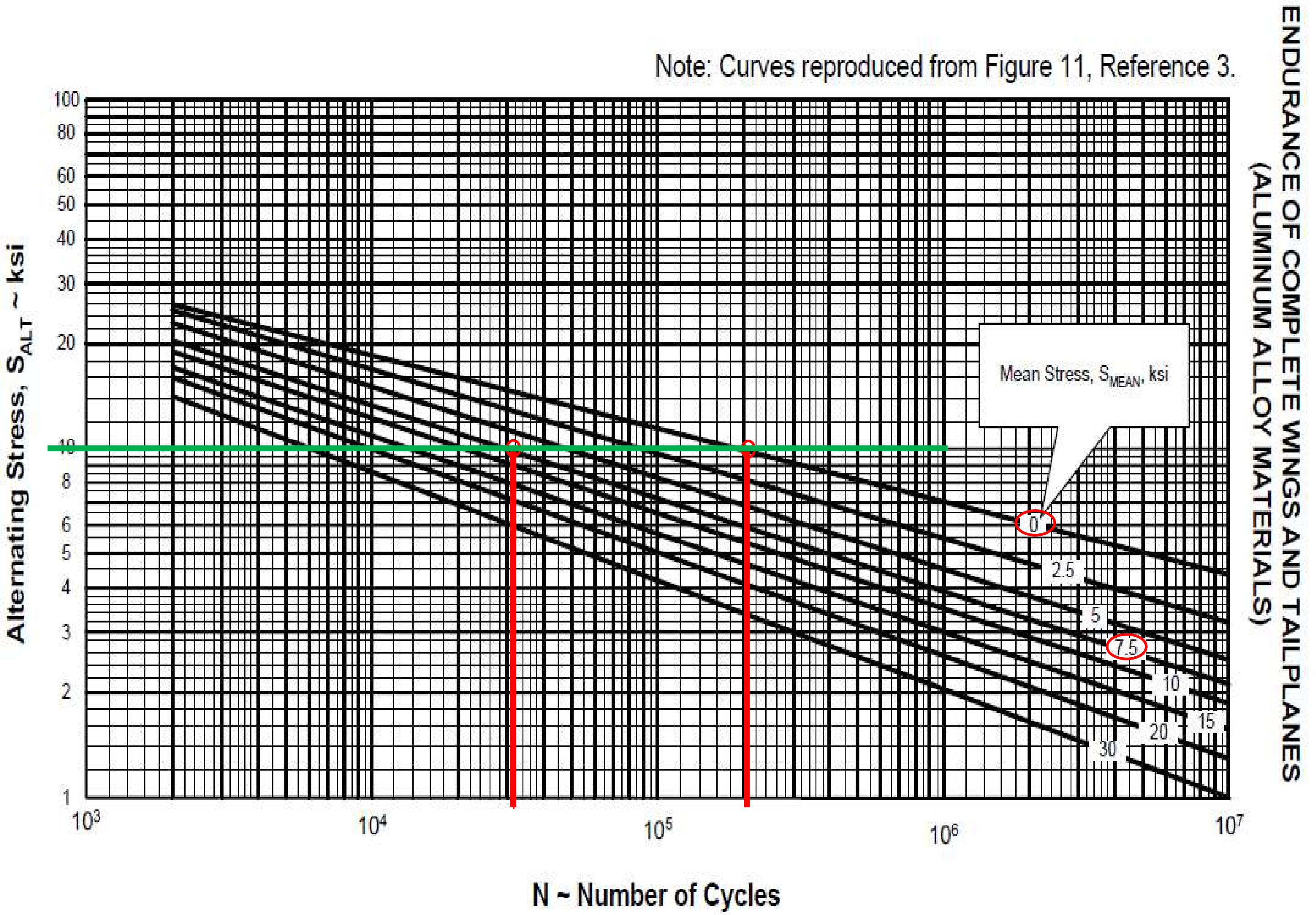
$b := -4.6$

$\sigma_a := 69\ \text{MPa} \qquad 10\ \text{ksi}$

$N = 2 \cdot 10^{5}$

When mean stress is 7.5ksi / 52MPa

Life ~ 32000



# Fatigue – Mean Stress

Exercise – 6E

- Calculate the fatigue life for the following stress reversal using different equations given.

$\sigma_{max} := 200\ MPa$

$\sigma_{min} := -80\ MPa$

$\sigma_{ty} := 280\ MPa$

$\sigma_{tu} := 450\ MPa$

$\sigma_f := 80\ MPa$

$A := 6.45 \cdot 10^{11}$

$b := -3.2$

$\left.\right\} \sigma_m = 0$

$$\frac{\sigma_a}{\sigma_{a\_eff}} + \frac{\sigma_m}{\sigma_y} = 1 \qquad Soderberg$$

$$\frac{\sigma_a}{\sigma_{a\_eff}} + \frac{\sigma_m}{\sigma_u} = 1 \qquad Goodman$$

$$\frac{\sigma_a}{\sigma_{a\_eff}} + \frac{\sigma_m}{\sigma_f} = 1 \qquad Morrow$$

$$\frac{\sigma_a}{\sigma_{a\_eff}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \qquad Gerber$$

$$\frac{\sigma_a}{\sigma_{a\_eff}} = \sqrt{\frac{1}{1 + \frac{\sigma_m}{\sigma_a}}} \qquad Smith - Watson - Topper$$

# Fatigue – Stress Concentrations

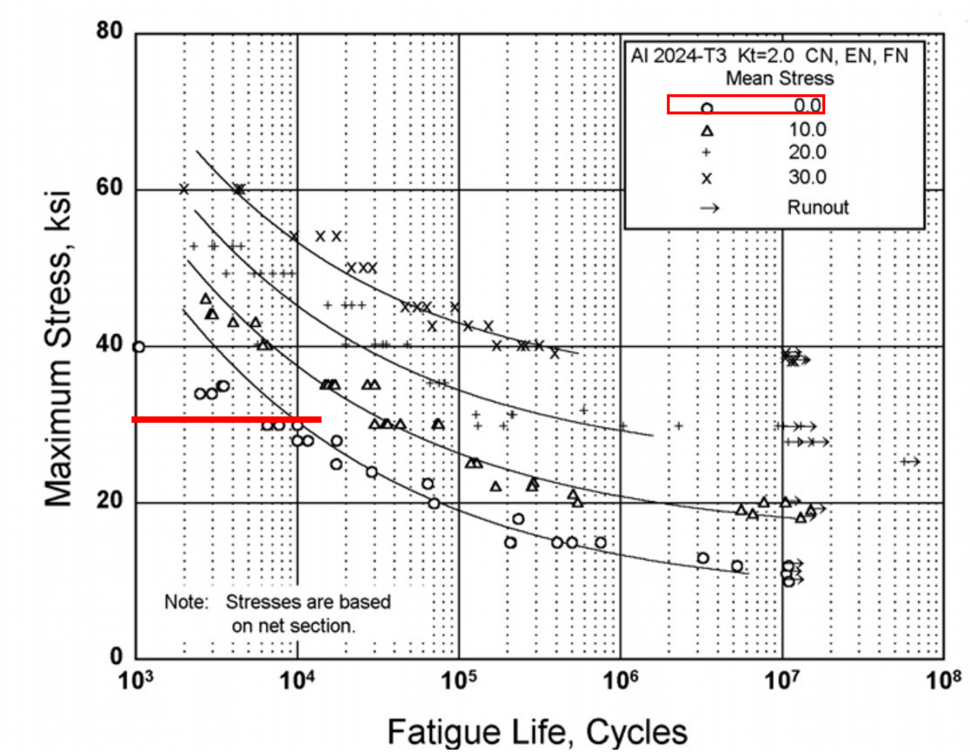
Exercise – 6F

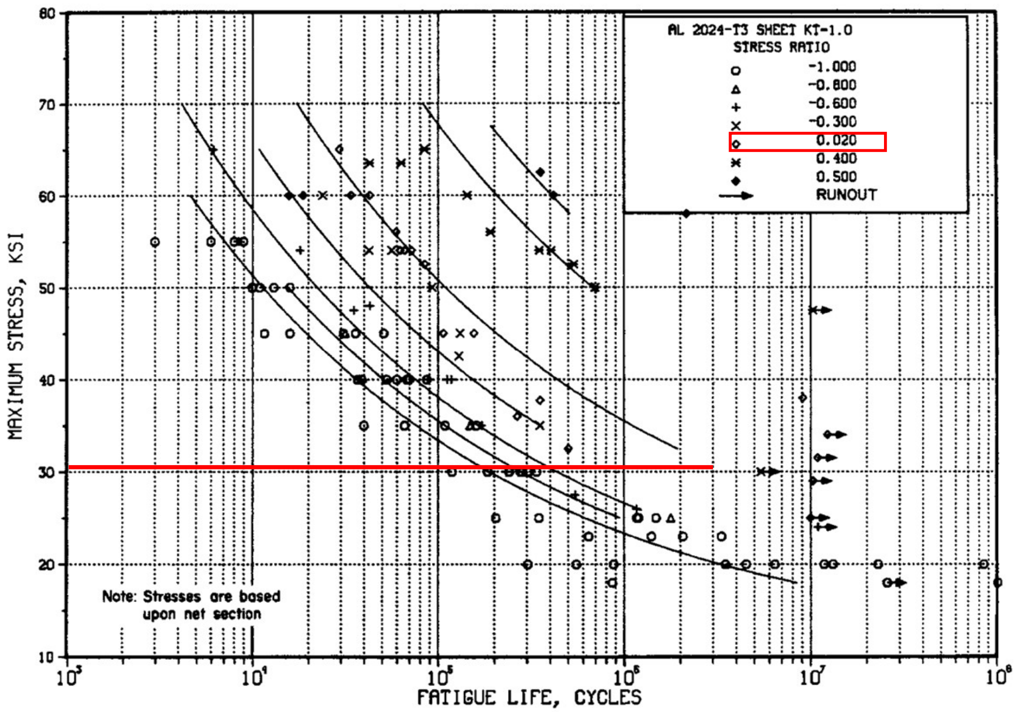
- Plate of width 40mm, thickness 2 mm has a hole of diameter 10mm. If the applied load is 10kN and the maximum stress at the edge of the hole is 415MPa. Calculate Net and Gross Stress Concentration Factors.

# Fatigue – Stress Concentrations

Exercise – 6F2

- Calculate the Notch Sensitivity Factor based on the two curves given below.



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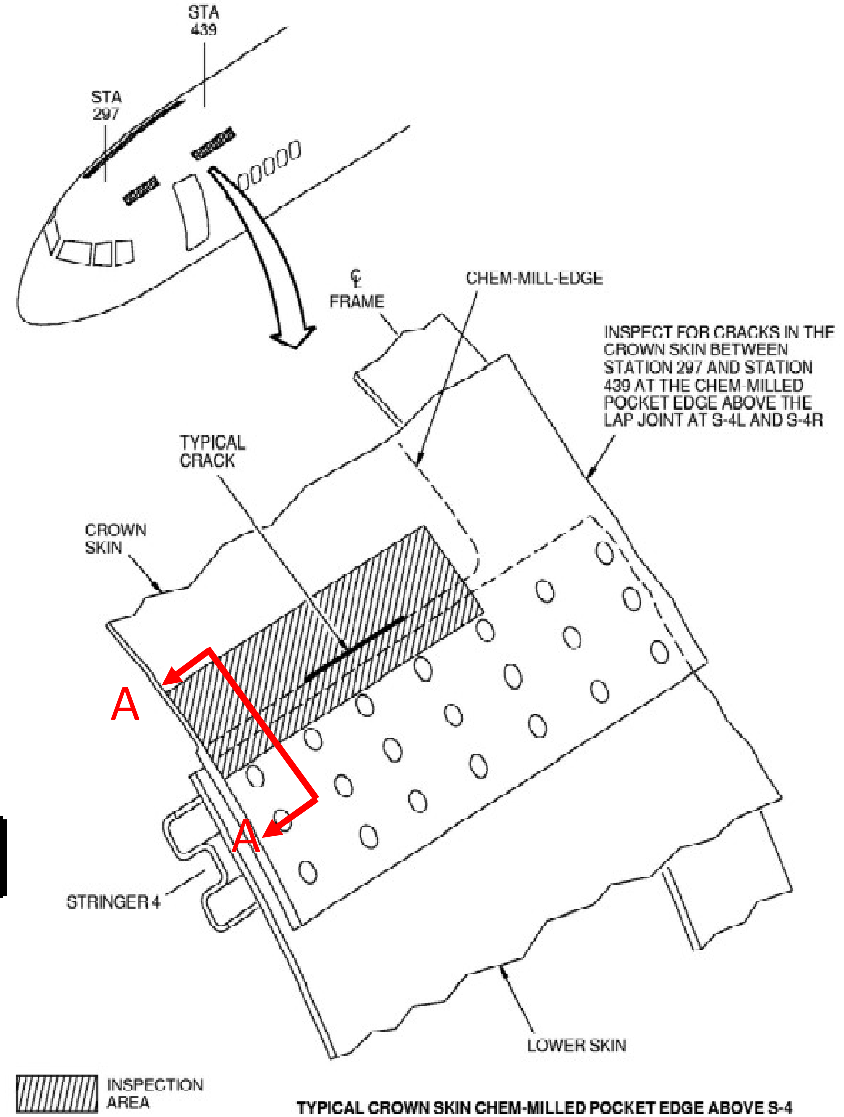
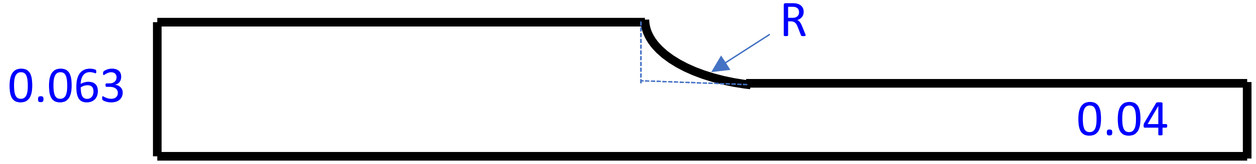
# Fatigue – Stress Concentrations

- Calculate the fatigue life for the fillet radius of skin thickness change as shown below. Fuselage radius of 74 in, Operating differential pressure of 8.6 psi. Fillet Radius of 0.05 in.
- Use a load reduction factor of 0.83 for hoop stress and 0.5 for local bending stress due to presence of frames.
- Use Aluminium fatigue S-N curve with A=4E+9, b=-3.2




TYPICAL CROWN SKIN CHEM-MILLED POCKET EDGE ABOVE S-4

Section A-A



# Fatigue – Stress Concentrations

Exercise – 6H

- Calculate Kt for the plate with a hole from previous problem if the reinforcing of outer dia 20mm and thickness of 4 is present.

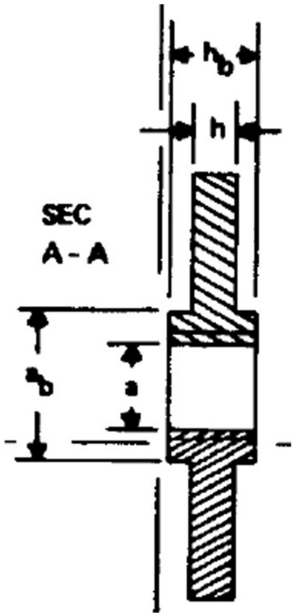
$a_b := 20\ \mathrm{mm}$

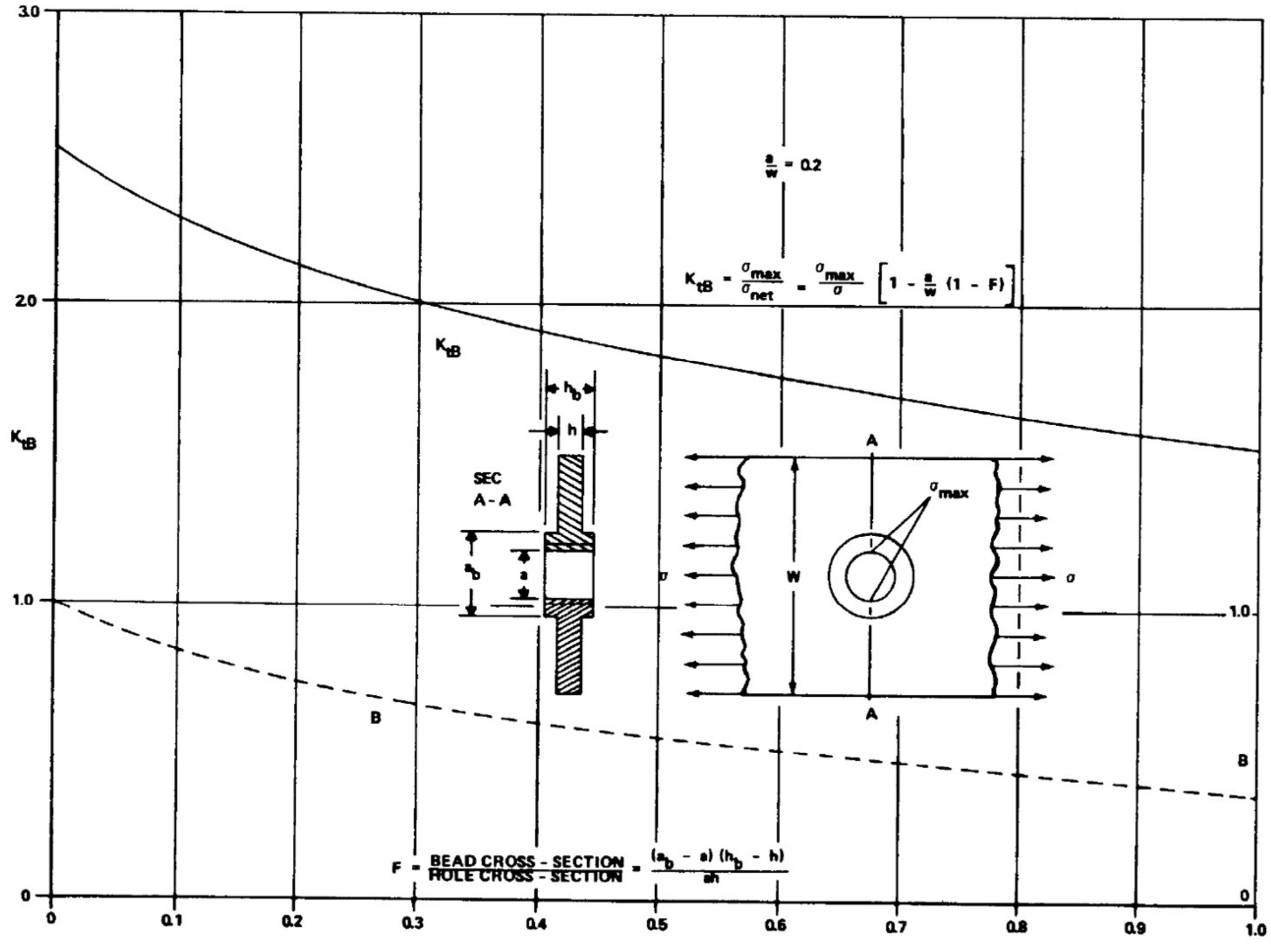
$a := 10\ \mathrm{mm}$

$h_b := 4\ \mathrm{mm}$

$h := 2\ \mathrm{mm}$

1



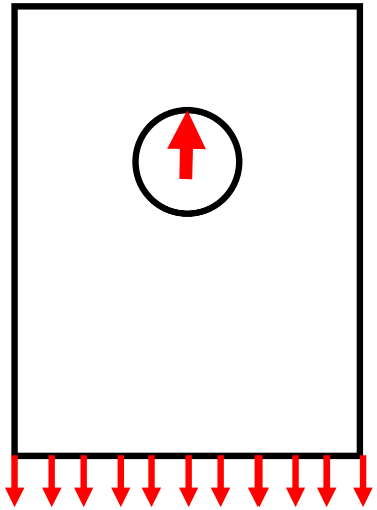




STRESS CONCENTRATION FACTOR, $K_{tB}$, FOR A TENSION PLATE WITH A BEADED HOLE.

# Fatigue of Joints

Exercise – 6I

- Calculate the maximum stress in the lug shown below.

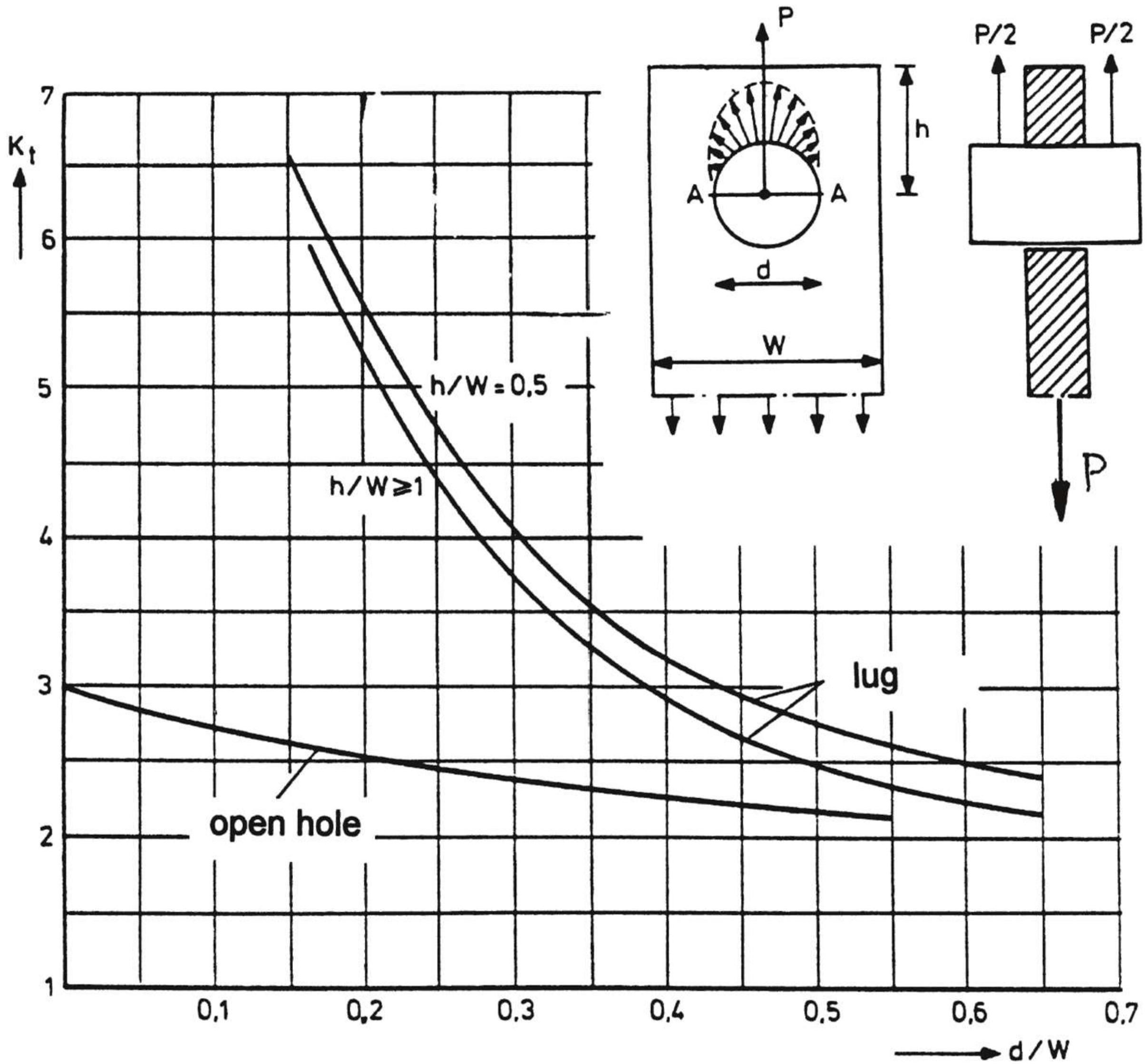


$w := 40\ \text{mm}$

$t := 2\ \text{mm}$

$d := 10\ \text{mm}$

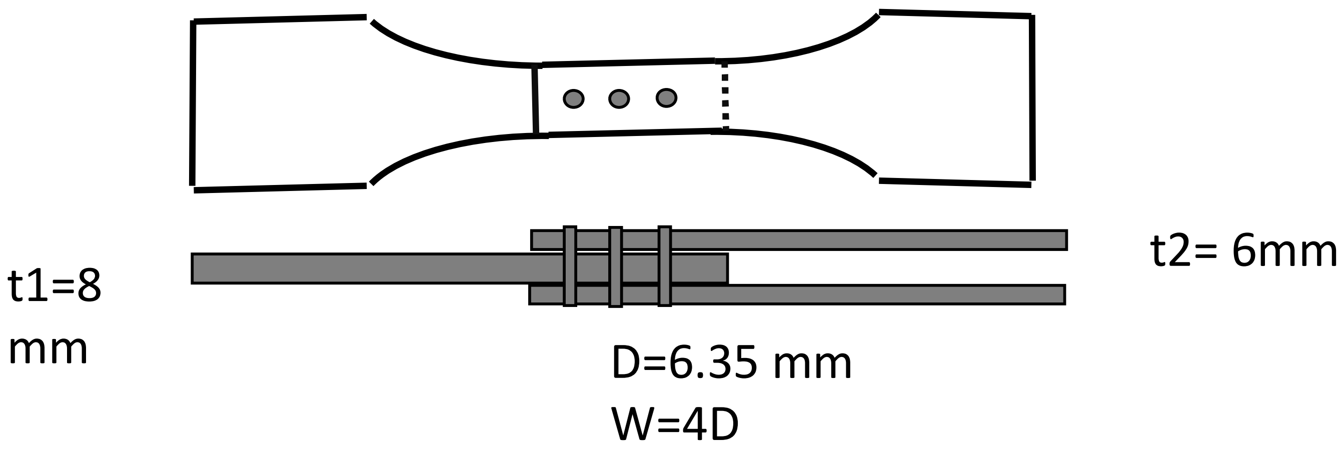
$P := 10000\ \text{N}$



# Fatigue of Joints

Exercise – 6J

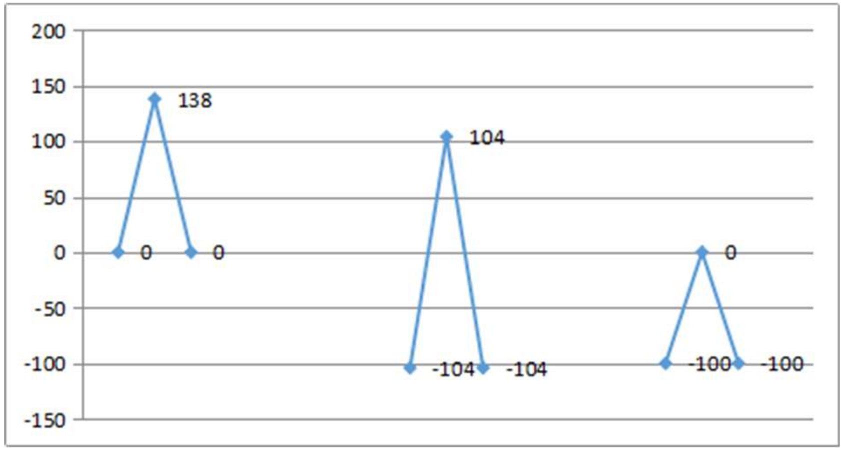
- Calculate maximum stress at the hole edges for both plates in the following configuration.
- Load = 10 kN
- Assume a bolt load of 40% for the end fasteners.



# Fatigue Spectrum – Cumulative Damage

Exercise – 6K

Calculate the cumulative damage and the life for the given blocked spectrum. Use goodman correction for mean stress with s_u=400 Mpa.




| Block | No of Repetitions | MIN | MAX |
|---|---|---|---|
| B1 | 30 | 0 | 138 |
| | 40 | -104 | 104 |
| | 60 | -100 | 0 |
| | | | |
| B2 | 20 | 0 | 138 |
| | 5 | -104 | 104 |
| | 30 | -100 | 0 |

$$A := 1 \cdot 10^{14}$$

$$b := -5$$

# Residual Strength – Notches

Exercise – 6L

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- Calculate the allowable stress that can be applied for the given materials and the stress concentration?

$$K_t \cdot \sigma_{app} = \sqrt{E \cdot \left(\left(\sigma_{max}\right) \cdot \left(\varepsilon_{max}\right)\right)}$$

$$K_t = 4$$

$E1 := 10800\ \text{ksi} = 74463\ \text{MPa}$

$\sigma_{tu1} := 63\ \text{ksi} = 434\ \text{MPa}$

$\varepsilon_{tu1} := 0.18$

$E2 := 10800\ \text{ksi} = 74463\ \text{MPa}$

$\sigma_{tu2} := 84\ \text{ksi} = 579\ \text{MPa}$

$\varepsilon_{tu2} := 0.08$

- Calculate the allowable Kt upto which the strength of the material is not reduced under static loading for the two materials.

# Residual Strength - Notches

Exercise – 6M

- Under uniaxial test a coupon yields at an applied stress of 330 MPa. At what applied stress does the a coupon with a notch (assuming plane strain condition at the root) will reach the same yield condition ?
- Use Poisson's ratio of 0.33
- At the root of the notch a pure bi-axial stress filed can be assumed

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu \cdot \sigma_x}{E} - \frac{\nu \cdot \sigma_y}{E}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

# Residual Strength – Fracture Toughness

Exercise – 6N

- What is the minimum crack length that can be tolerated by the below two materials when subjected to stress just below UTS.

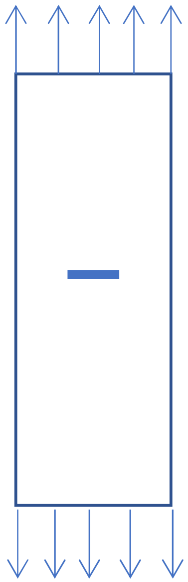
$$K = \sigma \cdot \sqrt{\pi \cdot a}$$

$K_1 := 45\ \mathrm{MPa} \cdot \sqrt{\mathrm{m}}$

$\sigma_{u1} := 450\ \mathrm{MPa}$

$K_2 := 50\ \mathrm{MPa} \cdot \sqrt{\mathrm{m}}$

$\sigma_{u2} := 1500\ \mathrm{MPa}$



# Residual Strength – Fracture Toughness

Exercise – 60

- Calculate the minimum thickness beyond which the crack in the material is always in plane strain for the two materials given.

$a := 3\ \text{mm}$ $\sigma := 100\ \text{MPa}$

$$t_{min} = 2.5 \cdot \frac{K_{1C}^{\ 2}}{\sigma_{ty}^{\ 2}}$$

$K_1 = 45\ \text{MPa} \cdot \sqrt{\text{m}}$

$\sigma_{ty1} = 320\ \text{MPa}$

$K_2 = 50\ \text{MPa} \cdot \sqrt{\text{m}}$

$\sigma_{ty2} = 1200\ \text{MPa}$

# Residual Strength – Crack

Exercise – 6P

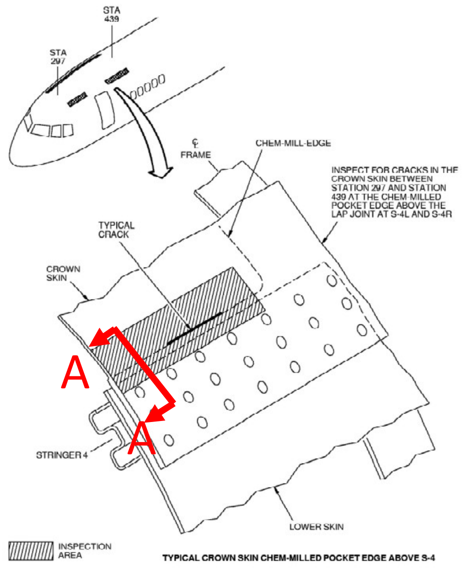
- Calculate the critical crack length for the fuselage skin for the pressurisation loading.

$\Delta p := 11\ \mathrm{psi}$

$D := 4\ \mathrm{m}$

$K1c := 31\ \mathrm{ksi} \cdot \sqrt{\mathrm{in}}$

$t := 1.5\ \mathrm{mm}$

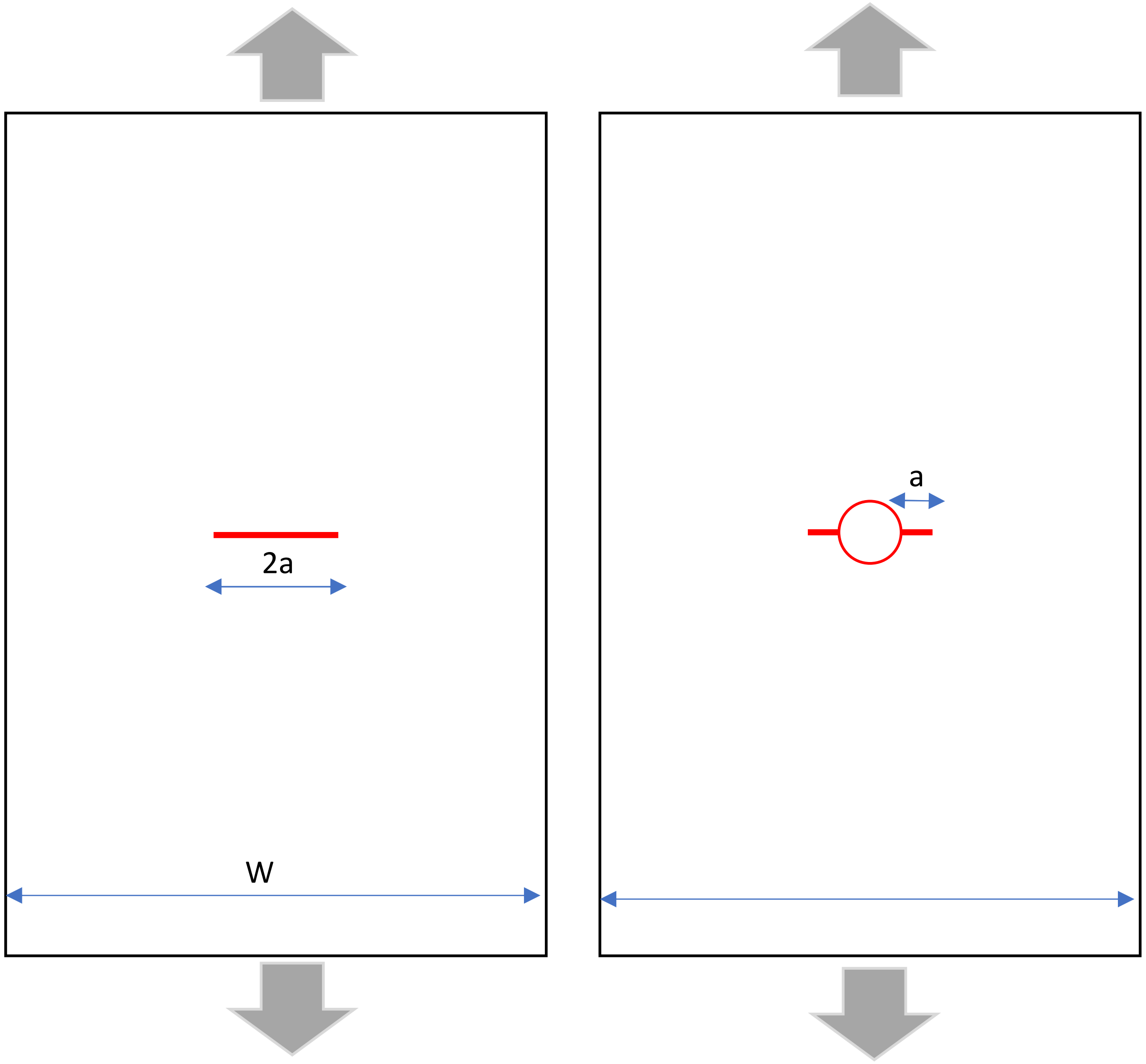


Exercise – 6Q

# Crack Propagation – SIF's

- Calculate SIF for a crack from a hole in the plate as shown below.
- Width of the plate = 100 mm , Hole diameter = 25 mm, for two crack lengths of 27 mm and 72 mm

a=13.5mm
&
36 mm




a = 1mm
&
23.5 mm

Measured from edge of the hole

# Crack Propagation

Exercise – 6R

- Crack growth data from a test from a crack length of 10 mm to 20 mm in a large plate (b =~1) is as follows.
- 0.5mm of first crack growth was for 2000 FC and last 0.5 mm growth was for 500 FC. Applied alternating stress was 100 MPa.
- Derive material properties for Paris Law.

- What would have been the total CP life from 10 mm to 20 mm ?
- What is the influence on the result of the number of increments in the calculation.

# Fuselage Skin

Calculate the crack propagation life from a detectable crack of 25mm to a critical crack length of 100mm for an applied stress of 0 to 35ksi (equivalent stress per cycle).

AR05-15

$$\frac{da}{dN} = C\left(\frac{\Delta K}{(1-R)^{1-m}}\right)^{n}$$

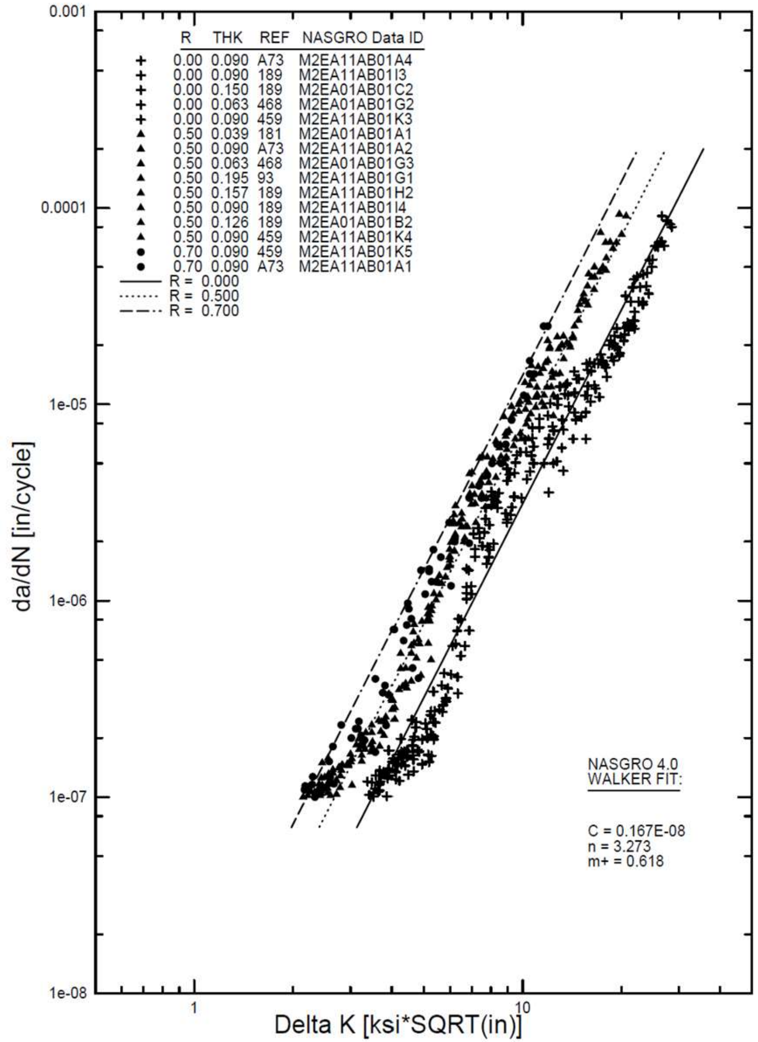
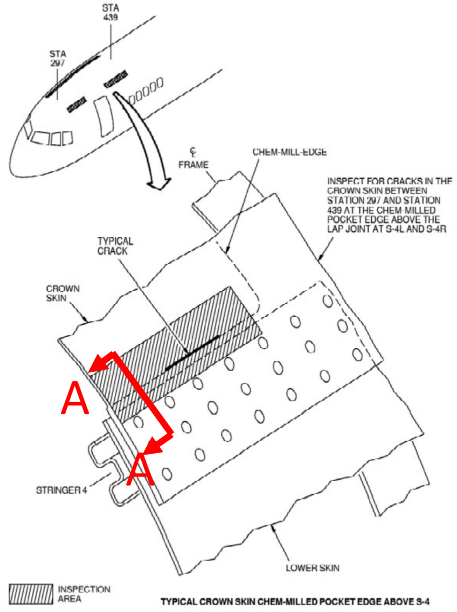


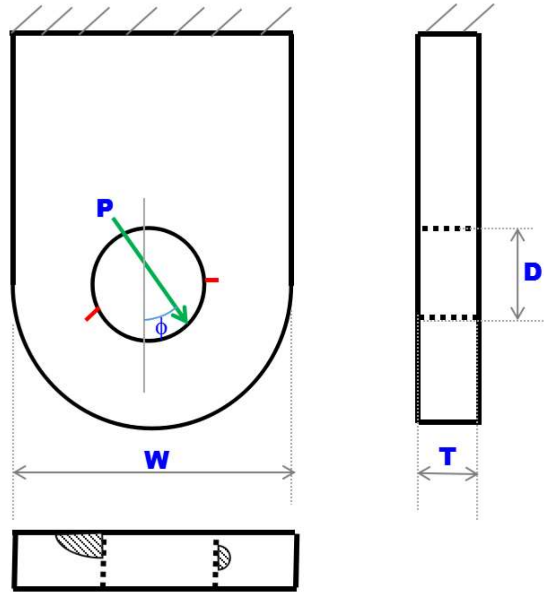

FIGURE B-5a. WALKER EQUATION FIT FOR 2024-T3 CLAD AND BARE SHEET, L-T (M2EA11AB1) WITH POSITIVE *R* VALUES



# Crack Propagation

Exercise – 6T

- Calculate the crack propagation life of Lug with the following details.
- w=120, D=55, t=20 mm
- Load for a representative case P=84 kN, factor from representative case to one cycle to stress=1.6
- Crack Growth Properties C =4E-9, m=3.2
- Use average K/K0 of 3.0 for corner crack and 2.0 for through crack on bearing stress
- Iterate the analysis if the single lug (SLP) is replaced with two lugs (MLP) with the net area in both cases being same.

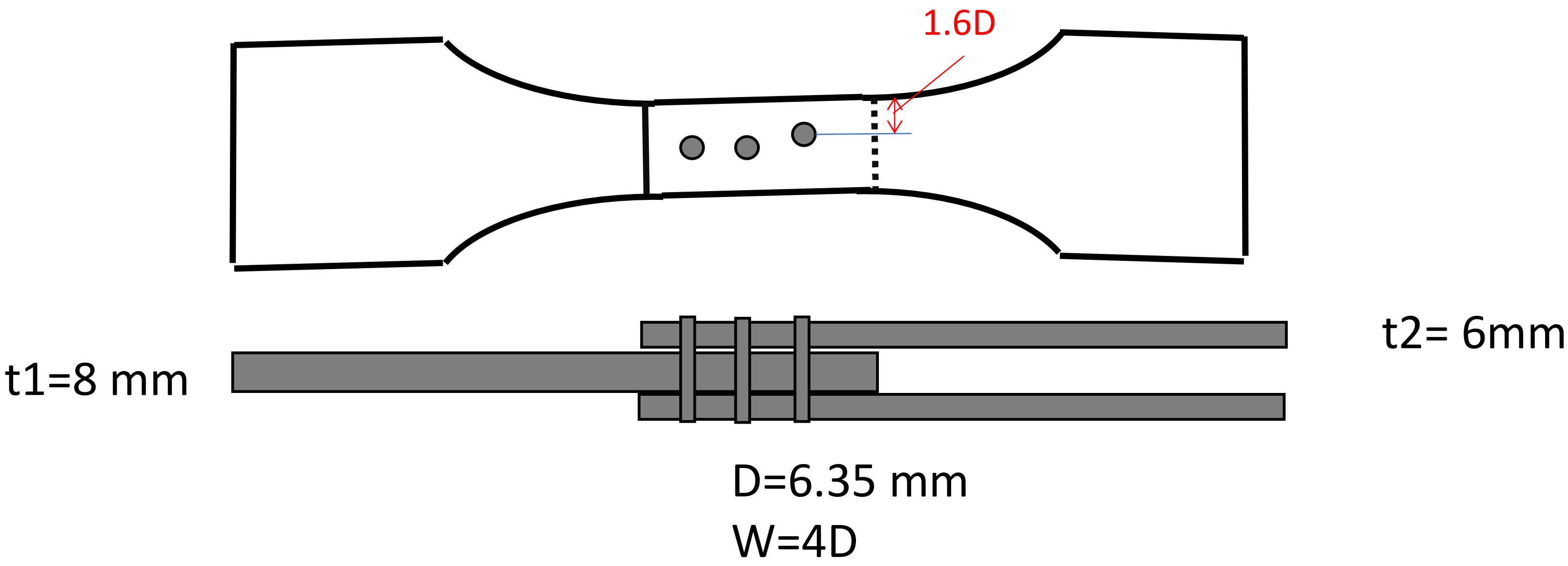


# Additional Exercises

# Production Issues

Exercise – XA

- Calculate maximum stress increase at the hole edges for the middle plate with the deviation of ED=1.6D compared to nominal of 2D
- Assume a bolt load of 40% for the end fasteners.
- Calculate fatigue life ratio is the slope of the SN curve was -5.4



Dimensions and Constants derived from Geometry

Loads and stresses

Material Properties

Others (depend on dimensions, Material properties)

Suffixes

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| | |
|---|---|
| $a$ | Crack Length |
| $A$ | Cross sectional area |
| $b$ | Width of a section |
| $c$ | Edge Distance / Offset |
| $d$ | Diameter |
| $D$ | Diameter |
| e | Edge Distance |
| $E$ | Young's Modulus |
| $F$ | Load |
| $f$ | Frequency , Hz |
| $G$ | Modulus of Rigidity |
| $g$ | Acceleration due to gravity |
| $h$ | Height of a section |
| $I$ | Area Moment of Inertia |
| $J$ | Polar Moment of Inertia |
| $k$ | Stiffness |
| $K$ | Stress Intensity Factor |
| $K_t$ | Stress Concentration Factor |
| $K_f$ | Fatigue Notch Factor |
| $L$ | Length |
| $MoS$ | Margin of Safety = RF - 1 |
| $M$ | Moment (Bending) |
| $m$ | Mass |
| $n$ | Number of ... |
| $n$ | Load Factor |
| $N$ | Number of Cycles |

| | |
|---|---|
| $p$ | Fastener Pitch |
| $P$ | Load |
| $q$ | Shear Flow |
| $Q$ | Product moment of area |
| $r$ | Radius / Radial distance |
| $RF$ | Reserve Factor = Allowable / Applied |
| $R$ | Load / Stress Ratio = Applied / Allowable |
| $t$ | Thickness |
| $T$ | Torque |
| $w$ | Width |
| $x$, $y$, $z$ | Distance long X, Y or Z axis |
| $\theta$ | Angle |
| $\sigma$ | Normal Stress |
| $\tau$ | Shear Stress |
| $q$ | Shear Flow |
| $\delta$ | Deflection / Elongation |
| $\upsilon$ | Poission's Ratio |
| $\varepsilon$ | Strain |
| $\nu$ | |
| $\omega$ | Angular Frequency |

| | |
|---|---|
| $alt$ | Alternating |
| $app$ | Applied |
| $all$ | Allowable |
| $bru$ | Bearing Ultimate |
| $br$ | bearing |
| $cy$ | Compressive Yield |
| $sh$ | Shear |
| $su$ | Shear Ultimate |
| $tu$ | Tensile Ultimate |
| $ty$ | Tensile Yield |
| $t$ | Tension |
| $xx$ | about X axis |
| $yy$ | about Y axis |
| $zz$ | about Z axis |
| $xy$ | in the XY plane |

$1\ \mathrm{psi} = 6895\ \mathrm{Pa}$

$1\ \mathrm{N} = 0.2248\ \mathrm{lbf}$

$1\ \mathrm{mph} = 0.447\ \frac{\mathrm{m}}{\mathrm{s}}$

$1\ \mathrm{knot} = 0.5144\ \frac{\mathrm{m}}{\mathrm{s}}$

$1\ \mathrm{mile} = 1609\ \mathrm{m}$

$1\ \mathrm{in} = 25.4\ \mathrm{mm}$

$1\ g_e = 9.8066\ \frac{\mathrm{m}}{\mathrm{s}^2}$

$1\ g_e = 32.174\ \frac{\mathrm{ft}}{\mathrm{s}^2}$

$1\ \mathrm{slug} = 32.174\ \mathrm{lb}$